Final – 250 points 600.488 Computational Geometry May 12, 1999

1. Please define each of the following terms:
   (a) distance metric
   (b) Delaunay triangulation
   (c) visibility between two points

2. 3-D Convex Hulls.
   (a) Why does a convex polyhedron with n vertices in $\mathbf{R}^3$ have $O(n)$ edges?
   (b) Very briefly describe an $O(n \log n)$-time algorithm for constructing the convex hull of n points in $\mathbf{R}^3$.

3. Arrangements.
   (a) Draw, as best you can, the arrangement of the following set of lines:

$$\begin{aligned} y &= x \\ y &= -x + 5 \\ y &= 1 \\ y &= x/2 + 2 \\ 3x + y &= 9 \end{aligned}$$

   (b) Briefly describe an $O(n^2)$-time method for constructing the arrangement of n lines in the plane.

4. Graph drawing.
   (a) Draw as best you can an orthogonal grid drawing of a graph G with 10 vertices and 16 edges.
   (b) Briefly describe an efficient algorithm for producing an orthogonal drawing of a (possibly unbalanced) binary tree, T. What is the worst-case running time and area bounds for this algorithm?

5. Voronoi Diagrams.
   (a) Draw, as best you can, the Voronoi diagram of the following set of points:

$$(1, 1)$$
$$(5, 2)$$
$$(2, 3)$$
$$(3, 3)$$
$$(1, 4)$$
$$(4, 5)$$

   (b) Sketch a fast method for constructing the Voronoi Diagram of n points in the plane. What is the worst-case running time of this method?

6. Visibility.

   (a) Draw, as best you can, the visibility graph of the following set of line segments (specified by their endpoints):

   $$[(0,0),(3,3)]$$
   $$[(1,2),(4,1)]$$
   $$[(3,2),(5,3)]$$
   $$[(2,4),(6,5)]$$
   $$[(-1,6),(4,5)]$$

   (b) Suppose you are given a set S of n non-intersecting line segments in the plane. Briefly describe an efficient method for computing the visibility graph of S. What is the running time of this method?

NOTE: For the remainder of this exam you may assume that you have a subroutine for any problem we discussed in class, provided you can correctly characterize its performance bounds.

7. Describe an efficient method for determining if a point q is inside or outside of an n-vertex simple polygon P in the plane. What is the running time of this method?

8. Suppose you are given two sets A and B, each containing n possibly-intersecting line segments in the plane. Describe an efficient method for determining if there is a single line L that separates A and B (so that the segments of A are entirely on one side of L and the segments of B are entirely on the other). What is the running time of this method?

9. Suppose you are given a rectangle map R that represents a flat tree-less valley, with its right boundary in the land of plenty and its left boundary in the land of hardship (so that R's two horizontal segments represent the sides of great mountains that cannot be climbed or crossed). Suppose further that your map is marked with n points, each of which represents a land mine. Describe what would be the safest path through R for you to follow in order to go from the land of hardship to the land of plenty, i.e., so that you go inside R from R's left end to R's right end while staying as far away from any land mine as possible. Explain how this path is related to a geometric structure we studied in class.

10. Suppose you are given an n-vertex simple non-convex polygon P. Define the *kernel* of P to be the set of all points q inside P such that q can see the entire polygon P. Describe an $O(n \log n)$-time method for computing the kernel of P.