CS 263 – Analysis of Algorithms

Homework 1, 50 Points
Due: Sunday, January 18, 11:00pm

This homework must be turned in electronically using the course GradeScope website.

Solutions **must** be submitted as a PDF file.

1. Show that randomized quicksort's best-case running time is $\Omega(n \log n)$.
2. Show how to use the accounting method to analyze PUSH and POP operations for a stack that is implemented using a dynamic array, assuming that the array doubles in size when its load factor exceeds 1 and the array halves in size when its load factor goes below 1/4.
3. Describe how to implement a union-find structure using extendable arrays, such that each array contains the elements that belong to a single set, instead of linked lists. Show how this solution can be used to process a sequence of m union-find operations on an initial collection of n singleton sets in time that is $O(n \log n + m)$.
4. Let A be a collection of objects. Describe an efficient method for converting A into a set. That is, remove all duplicates from A. What is the running time of this method?
5. Show that randomized quick-sort runs in $O(n \log n)$ time with probability at least $1 - 1/n^2$. **Hint**: Use the Chernoff bound that states that if we flip a coin k times, then the probability that we get fewer than $k/16$ heads is less than $2^{-k/8}$.