# Rooted Cycle Bases

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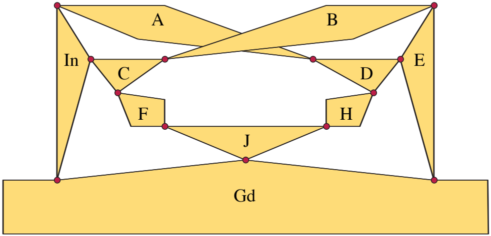
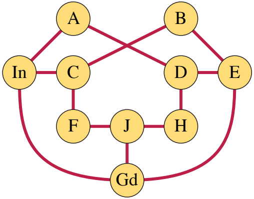
# Linkages

Systems of rigid bodies connected by hinges

Often with designated ground body (fixed in place)
and input body (where force input causes system to move)



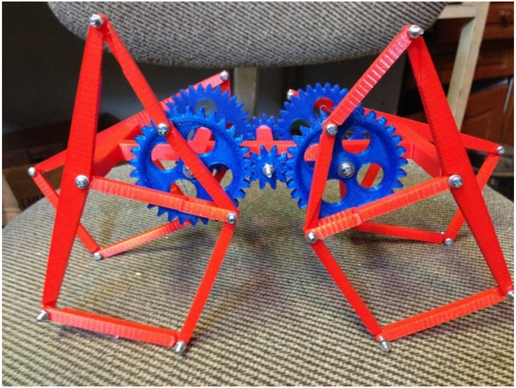




Connectivity may be described by a graph
with vertices = bodies and edges = hinges

Simplifying assumptions: system lies in $\mathbb{R}^2$ (crossings allowed);
input is attached to ground (so its motion is purely rotational)

# Complex linkages can have complex motions





Applications include walking robots, fold-away sofabeds, vehicle suspensions, low-clearance doors, therapeutic exoskeletons, ...

# Our piece of a larger puzzle

Goal:
design linkage that achieves some desired motion

Sub-goal:
Analyze motion of a linkage

Sub-sub-goal:
Set up independent equations for the motion of a linkage

Combinatorial abstraction:
Find independent input–ground paths in the graph of a linkage

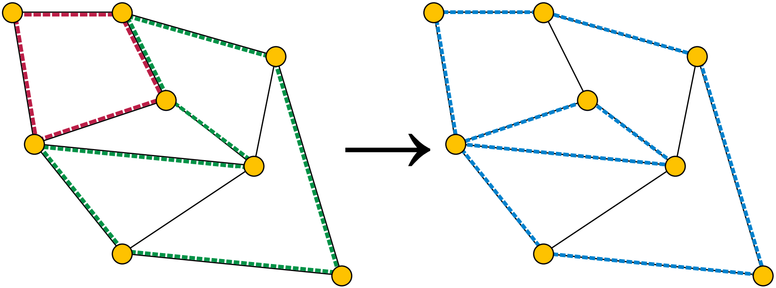
# Independence of cycles in graphs

Cycle space: a vector space over $\mathbb{Z}_2$

Elements = sets of edges with even degree at all vertices

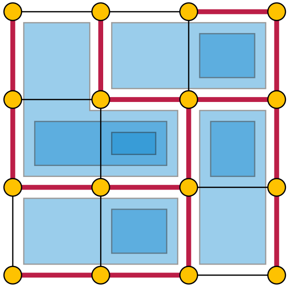
Vector sum = symmetric difference of edge sets

Scalar product = trivial

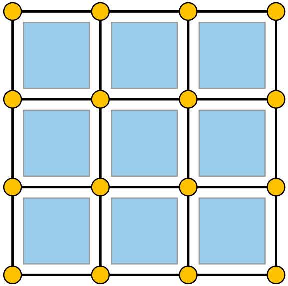


# Cycle bases

Cycle basis = basis of the cycle space =
maximal independent set of cycles



Fundamental cycle basis:
cycles are spanning tree
paths + one edge



In planar graphs,
bounded faces form
a cycle basis

# Greed is good

Avarice, from The Dunois Hours, France, ca. 14401450. Public domain image “The Dunois Hours Avarice” on Wikimedia commons.

Independence in a vector space forms a matroid ⇒ minimum weight cycle bases can be found by a greedy algorithm

For each candidate cycle, sorted by weight, test if independent and, if it is, include in basis

The difficult part: finding a small set of candidate cycles

When all weights positive, possible in polynomial time

# What kind of cycle basis do we need?

Paths from linkage input to ground ⇔ cycles that all pass through a "root edge" incident to input and ground

Rooted graph: undirected graph + one chosen root edge

Rooted cycle basis: all cycles in the basis include the root edge

Higher cycle length gives more complex equations to solve, so we want a minimum weight rooted cycle basis

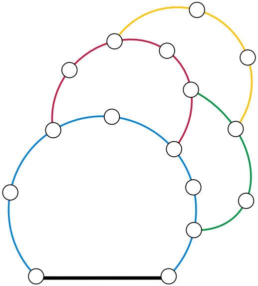
Plate to: H. J. Ruprecht, wand-atlas, ed. 3, Dresden, c. 1850, no. 15. Public domain image "A plant root cut to show growth rings, wood cells in longitu Wellcome V0044550" on Wikimedia commons.

# Warm-up result

Rooted cycle basis exists if and only if graph has one non-trivial 2-vertex-connected component and it contains the root edge

Proof uses ear decomposition:

- Sequence of simple paths
- First path = root edge
- Endpoints of remaining paths lie on earlier paths
- Interior vertices of each path are disjoint from earlier paths

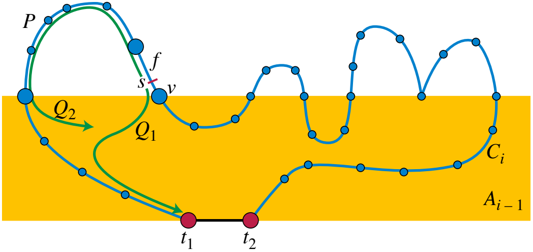


Proof idea: Extend each ear to rooted cycle through previous ears

# The main result

Polynomial-time construction of min-weight rooted cycle basis

Main idea: new edges of each greedy basis cycle form one path; these paths form an ear decomposition




Part of case analysis showing that a cycle with $> 1$ new-edge paths cannot be the greedy choice

Corollary: shortest rooted cycles through each edge form a valid candidate set

Use Suurballe's algorithm to find these cycles

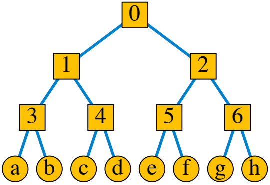
# Breaking ties

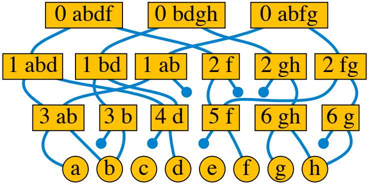
Correctness proof is only valid when all cycle lengths are distinct

Small random perturbation works but requires randomness

Perturbation by small powers of two works
but requires too many bits of numerical precision

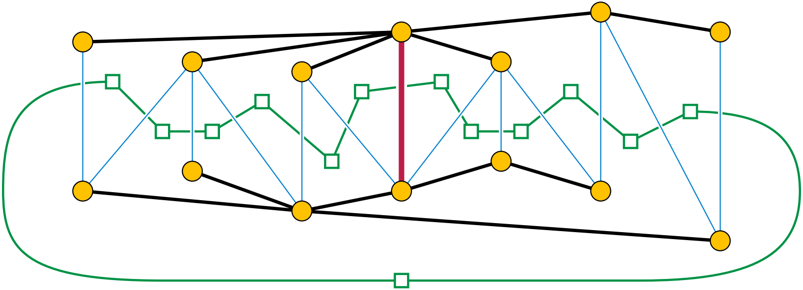
Data structure for simulating power-of-two perturbation
incurs only logarithmic slowdown



# Additional results

Finding a fundamental rooted cycle basis is NP-complete
(planar dual of finding a rooted Hamiltonian cycle)



But can be solved in fixed-parameter time for (nonplanar) graphs
of small treewidth or clique-width using Courcelle's theorem

# Conclusions

New type of cycle basis

Motivated by applications in linkage analysis

With a polynomial time optimization algorithm

Also useful for other applications?