CS 261 — Spring 2024 — Final exam (not comprehensive)

Name:

UCI email:

Do not open this exam until told to start by the instructor.

This is a closed book, closed note exam. Calculators or other electronic devices are not allowed.

Please write your answers ONLY on the front side of each page.
Answers written elsewhere will not be scanned and will not graded.

You may use the back sides of the pages as scratch paper.
Do not unstaple the pages.

1. (15 points) Let T be a splay tree with n keys in it, the numbers from 1 to n. If we set the weight of each key i to $w_i = 1/i$, what is the amortized time to access key i in T, according to the weighted amortized analysis of splay trees? Give your answer in O-notation, using i and n. (Note: Other weights may be better, but analyze these weights. You may find it helpful to use the harmonic series in the form $\sum_{i=1}^{n} 1/i = \ln n + O(1)$, where ln is the natural logarithm.)

2. (15 points) Suppose that we wish to maintain a collection of pairs of numbers (x_i, y_i), and answer "range average" queries: given a query interval $[L, R]$, find the average of y_i among the pairs (x_i, y_i) with $L \leq x_i \leq R$. Describe a way of augmenting the nodes of a binary search tree (using the numbers x_i as keys) with two additional numbers that would allow you to compute this average query in time $O(\log n)$, and describe in English (not pseudocode) how to use these numbers to answer the query.

3. (15 points) Find a sequence of the seven numbers from 1 to 7 whose Cartesian tree has four leaves.

4. (15 points) Suppose that we have implemented a binary tree structure using fully persistent (path copying) zippers, and that we perform the following sequence of operations:

   - Create a new binary tree with its finger pointing to one root node, x
   - Add y as a left child of x (leaving the finger at x)
   - Add z as a right child of x (leaving the finger at x)
   - Move the finger from x to y

   How many copies of node x, node y, and node z will be created by these operations? (Your answer should be three numbers: the number of copies of x, the number of copies of y, and the number of copies of z, including in each case the first copy of each node. You may find it helpful to draw the nodes created by these operations, but that is not required for your answer.)

5. (15 points) List the suffixes of the string eerier$ (including the suffix that contains only the end-of-string character), in the order that they would be listed in a suffix array. Then, also give the suffix array itself, as a list of start positions (without added longest common prefix information). As in the lecture notes, number string positions starting with 0, sort the end-of-string character $ first, and then use the alphabetical ordering $ < e < i < r. (Eerier is an English word meaning "more strange and frightening".)

6. (15 points) Draw a suffix tree for the string eerier$. The children of each node should be drawn in sorted order.

7. (15 points) Suppose that two nodes x and y of a tree have lowest common ancestor z, and that each node is labeled by its distance $x.d$, $y.d$, and $z.d$ from the root of the tree (an integer number of edges). Give a formula involving these quantities for the distance from x to y.

8. (15 points) Suppose we have a union-find data structure with eight elements, a, b, c, d, e, f, g, h, and i, forming two sets {a, b, c, d} and {e, f, g, h, i}, with the following structure:

   - In the first set, a has parent b; b and c have parent d; d is a root.
   - In the second set, e has parent f; f, g, and h have parent i; i is a root.

   (a) Draw the structure that results after the operation union(i,d).

   (b) Draw the structure that results from performing union(i,d) (as in part a) and then find(a).