Vector Space Scoring

Introduction to Information Retrieval

INF 141

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Content adapted from Hinrich Schütze
http://www.informationretrieval.org

Corpus-wide statistics

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  - Define: The total number of occurences of the term in the entire corpus

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- Document Frequency, df
  - Define: The total number of documents which contain the term in the corpus

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Word	*Collection Frequency*	*Document Frequency*
insurance	10440	3997
try	10422	8760

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- How do we use df?

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    - more commonly it is: $$idf_t = log\left(\frac{|corpus|}{df_t}\right)$$

TF-IDF Examples

$$idf_t = log\left(\frac{|corpus|}{df_t}\right) \qquad idf_t = log_{10}\left(\frac{1,000,000}{df_t}\right)$$

$term$	df_t	idf_t
$calpurnia$	1	6
$animal$	10	4
$sunday$	1000	3
fly	$10,000$	2
$under$	$100,000$	1
the	$1,000,000$	0

TF-IDF Summary

- Assign tf-idf weight for each term t in a document d:

$$tfidf(t,d) = (1 + log(tf_{t,d})) * log\left(\frac{|corpus|}{df_{t,d}}\right)$$

  - Increases with number of occurrences of term in a doc.
  - Increases with rarity of term across entire corpus
  - Three different metrics
    - term frequency
    - document frequency
    - collection/corpus frequency

Now, real-valued term-document matrices

- Bag of words model
- Each element of matrix is tf-idf value

	Antony and Cleopatra	*Julius Caesar*	*The Tempest*	*Hamlet*	*Othello*	*Macbeth*
Antony	13.1	11.4	0.0	0.0	0.0	0.0
Brutus	3.0	8.3	0.0	1.0	0.0	0.0
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Cleopatra	17.7	0.0	0.0	0.0	0.0	0.0
mercy	0.5	0.0	0.7	0.9	0.9	0.3
worser	1.2	0.0	0.6	0.6	0.6	0.0

Vector Space Scoring

- That is a nice matrix, but
  - How does it relate to scoring?
  - Next, vector space scoring

Vector Space Model

- Define: Vector Space Model
  - Representing a set of documents as vectors in a common vector space.
  - It is fundamental to many operations
    - (query,document) pair scoring
    - document classification
    - document clustering
  - Queries are represented as a document
    - A short one, but mathematically equivalent

Vector Space Model

- Define: Vector Space Model
  - A document, d, is defined as a vector: $\vec{V}(d)$
    - One component for each term in the dictionary
    - Assume the term is the tf-idf score

$$\vec{V}(d)_t \quad = \quad (1 + log(tf_{t,d})) * log\left(\frac{|corpus|}{df_{t,d}}\right)$$

- A corpus is many vectors together.
- A document can be thought of as a point in a multi-dimensional space, with axes related to terms.

Vector Space Model

- Recall our Shakespeare Example:

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$\vec{V}(d_6)_7$

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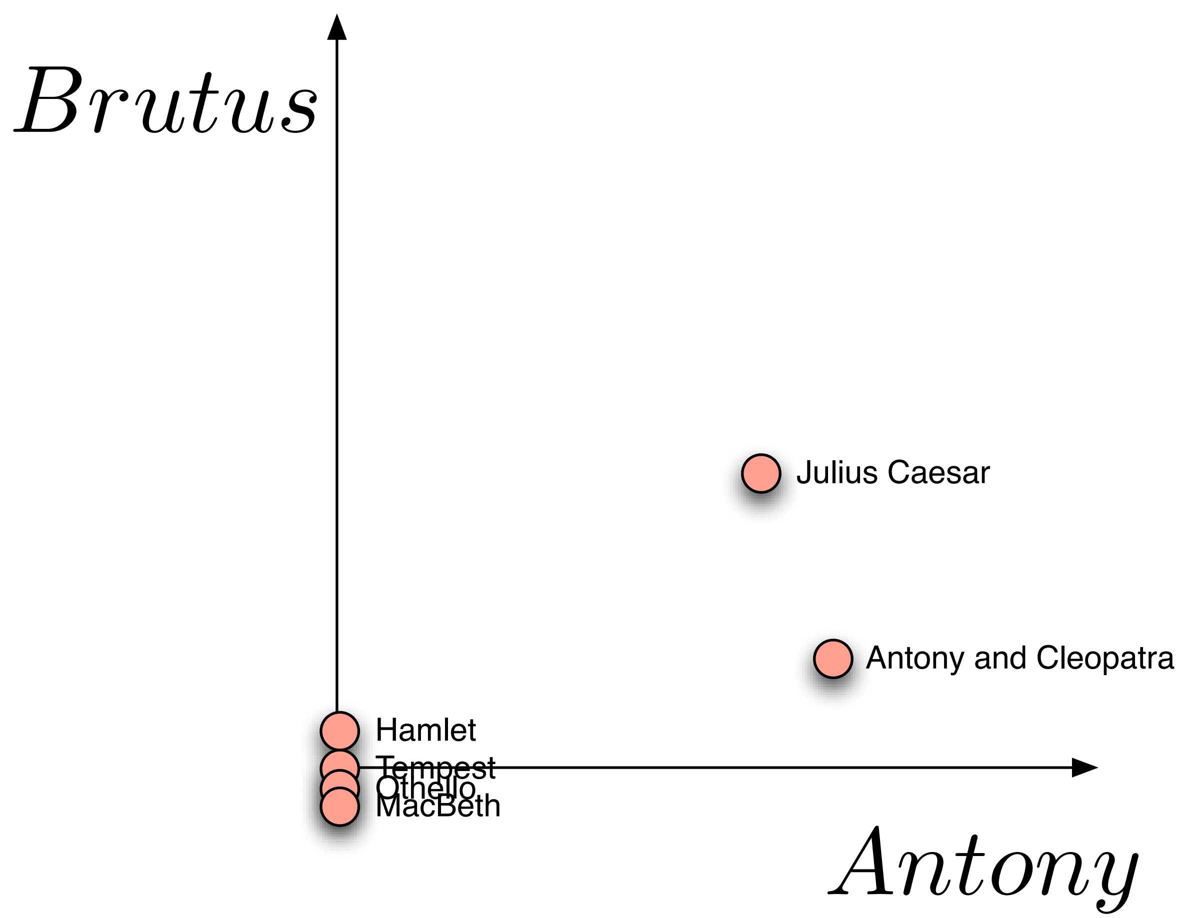

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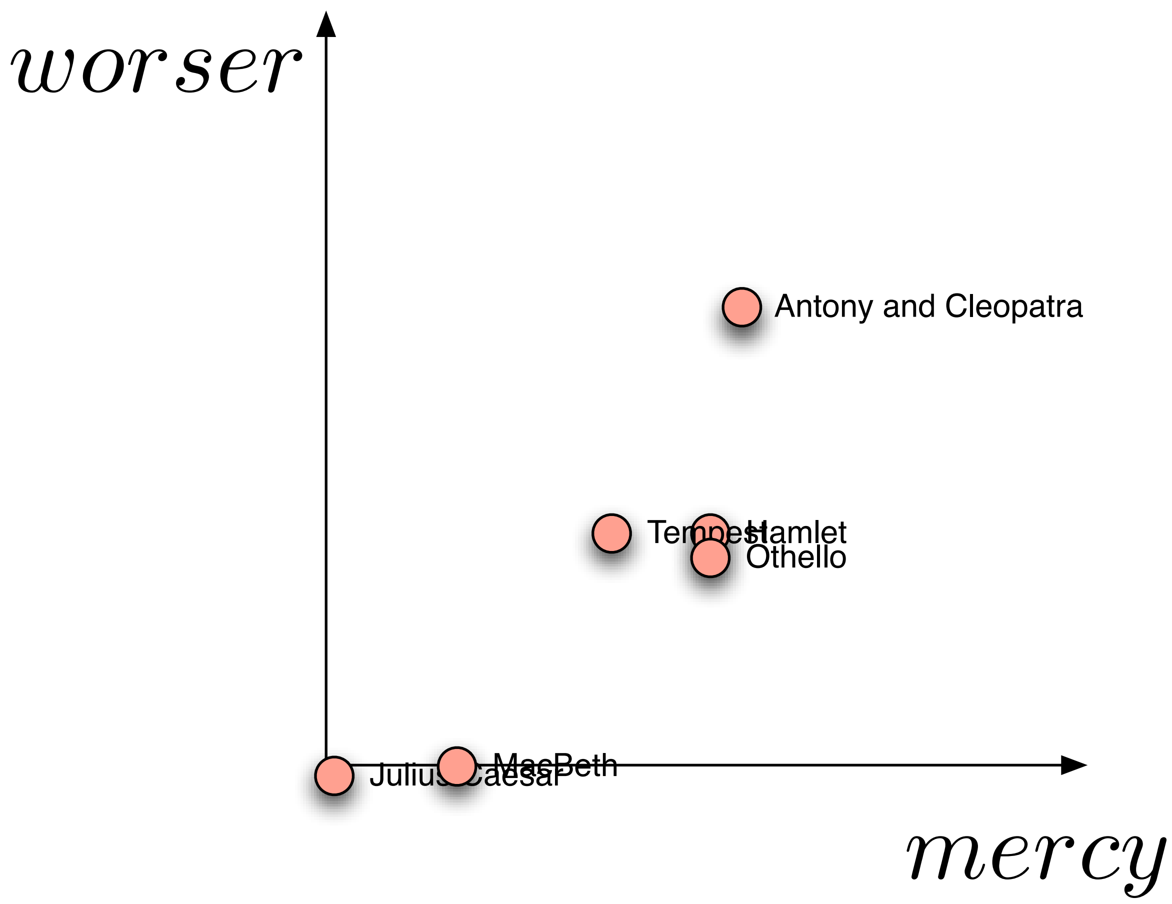
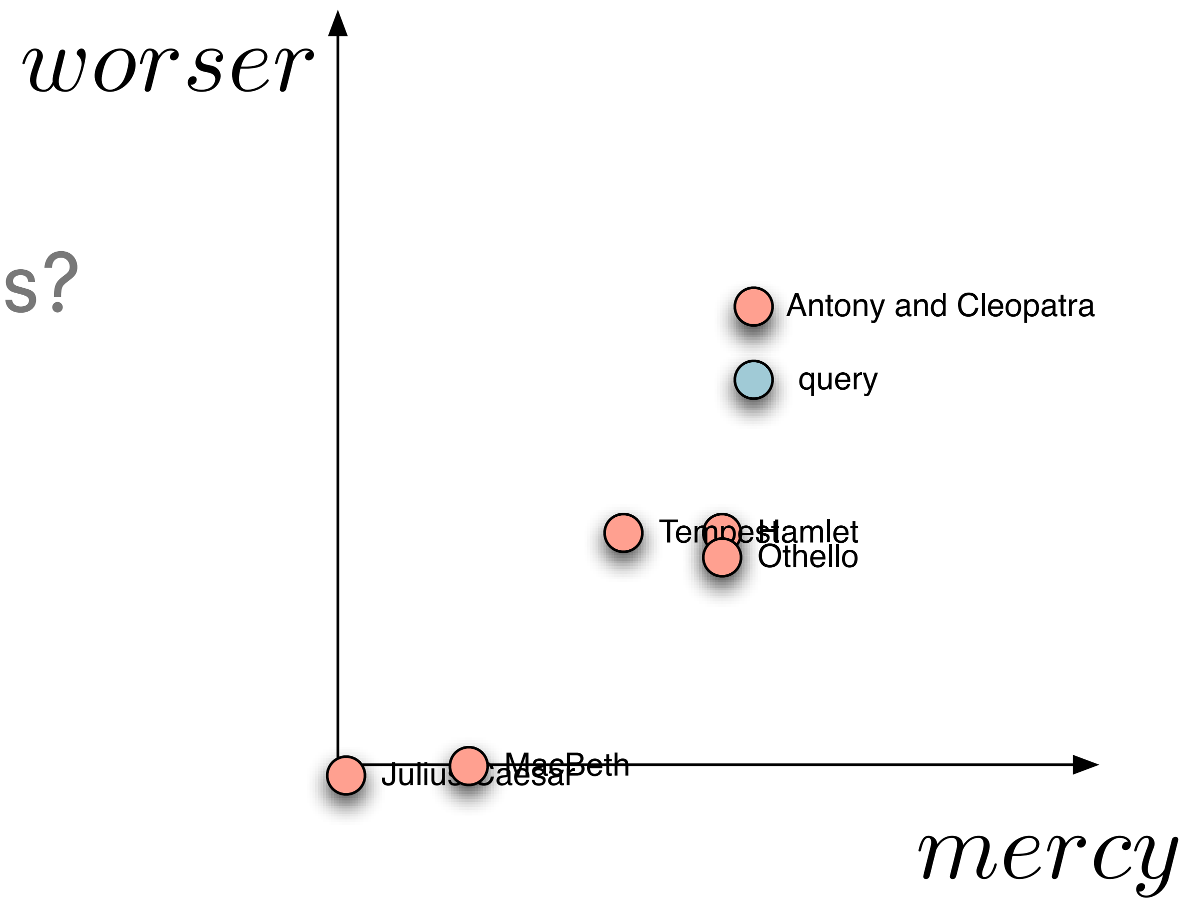


Vector Space Scoring

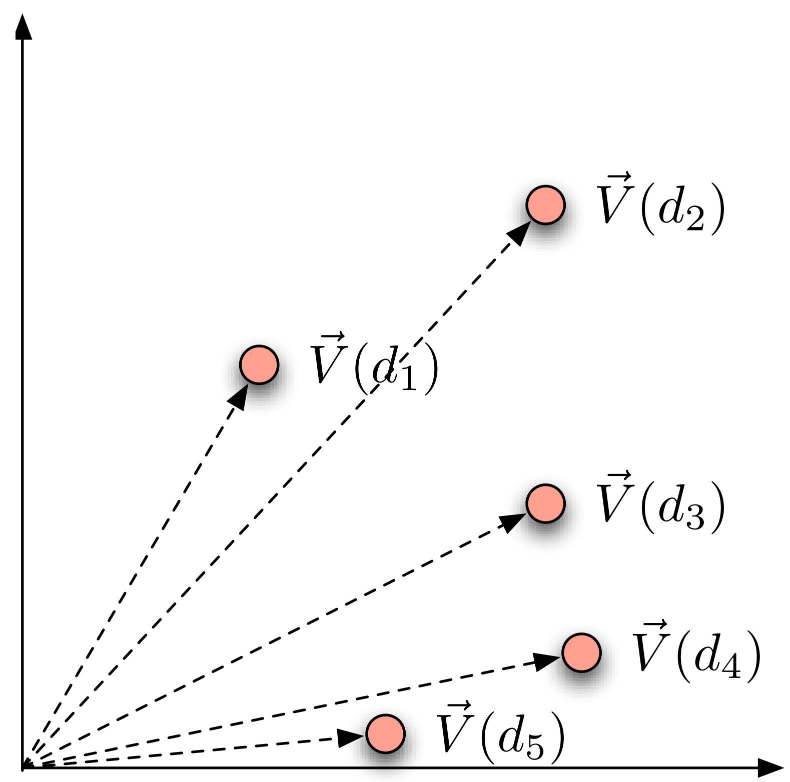
Query as a vector

- So a query can also be plotted in the same space
  - "worser mercy"
  - To score, we ask:
    - How similar are two points?
  - How to answer?



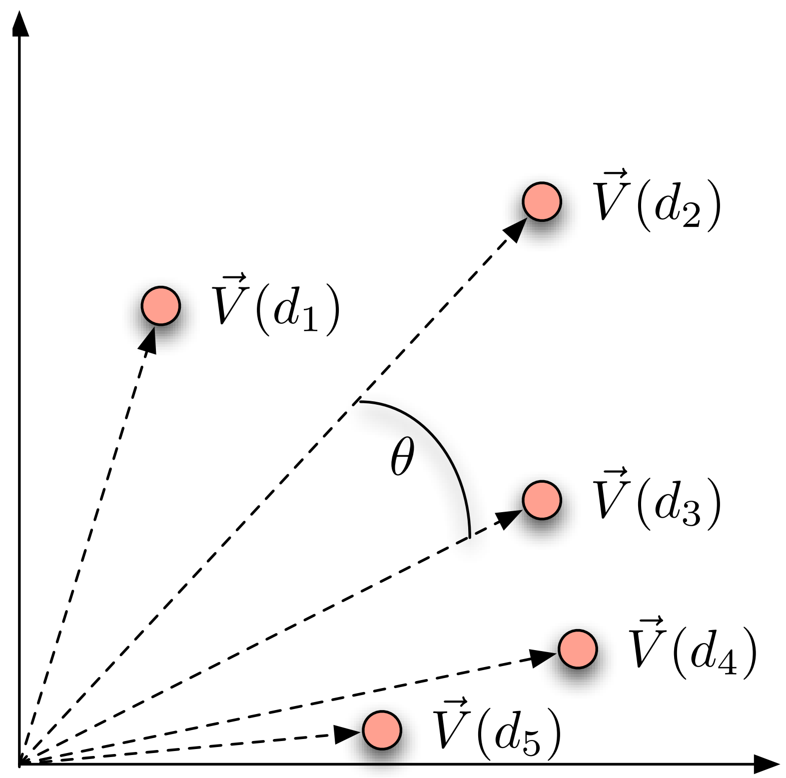
Score by magnitude

- How to answer?
  - Similarity of magnitude?
    - But, two documents, similar in content, different in length can have large differences in magnitude.



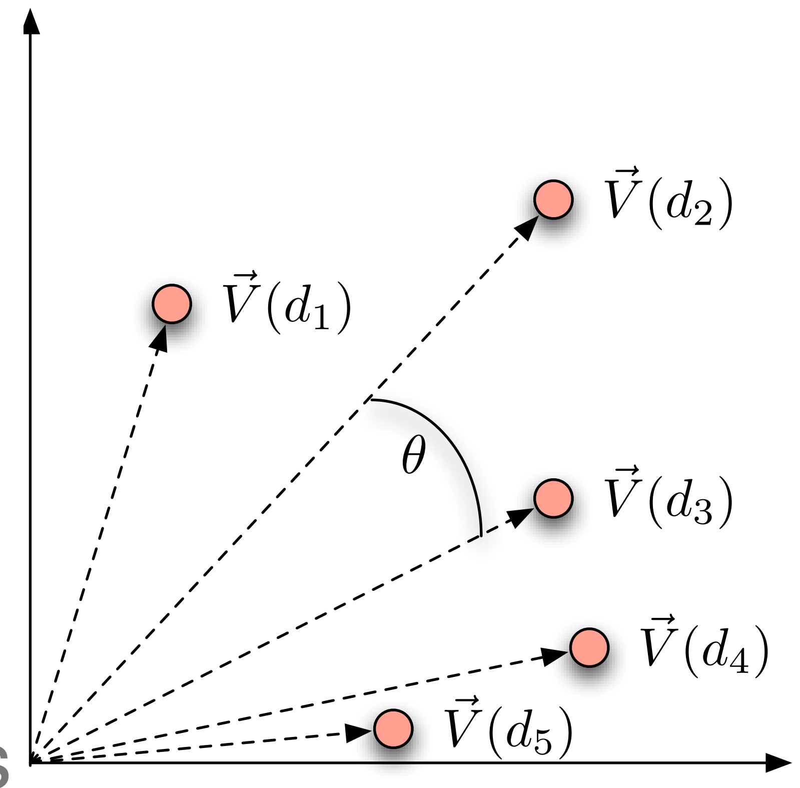
Score by angle

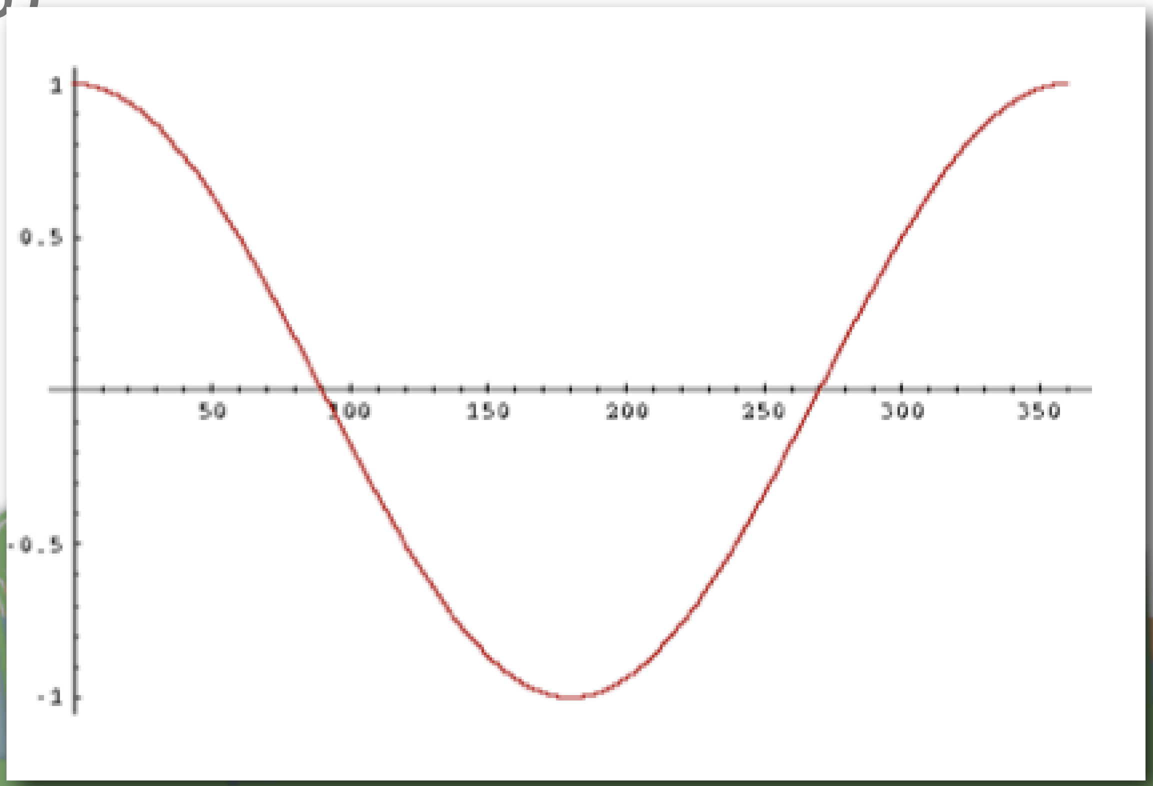
- How to answer?
  - Similarity of relative positions, or
  - difference in angle
    - Two documents are similar if the angle between them is 0.
    - As long as the ratios of the axes are the same, the documents will be scored as equal.
    - This is measured by the dot product



Score by angle

- Rather than use angle
  - use cosine of angle
  - When sorting cosine and angle are equivalent
  - Cosine is monotonically decreasing as a function of angle over (0 ... 180)





Big picture

- Why are we turning documents and queries into vectors
  - Getting away from Boolean retrieval
  - Developing ranked retrieval methods
  - Developing scores for ranked retrieval
  - Term weighting allows us to compute scores for document similarity
  - Vector space model is a clean mathematical model to work with

Big picture

- Cosine similarity measure
  - Gives us a symmetric score
    - if d_1 is close to d_2, d_2 is close to d_1
  - Gives us transitivity
    - if d_1 is close to d_2, and d_2 close to d_3, then
    - d_1 is also close to d_3
  - No document is closer to d_1 than itself
  - If vectors are normalized (length = 1) then
    - The similarity score is just the dot product (fast)

Queries in the vector space model

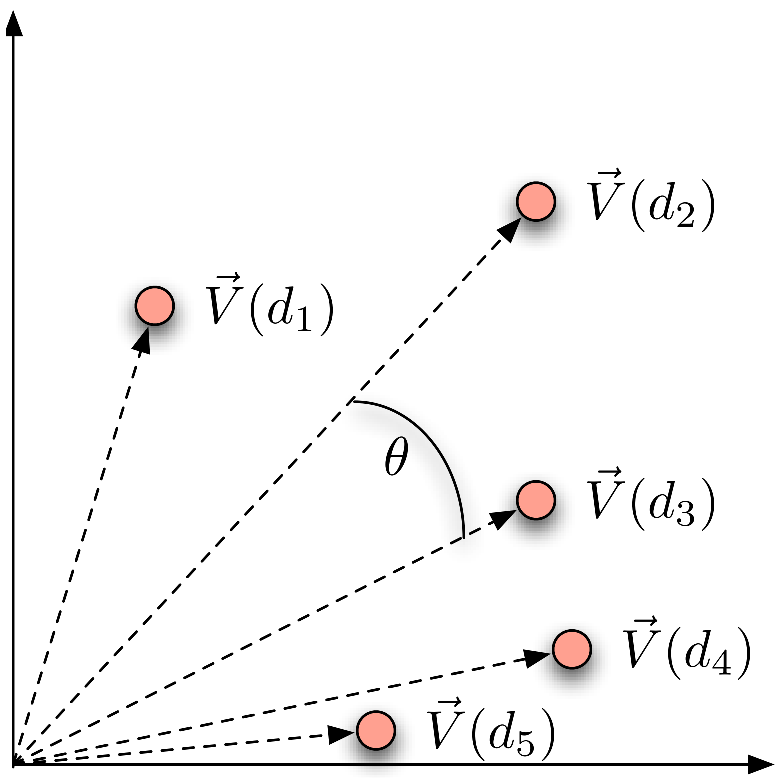
- Central idea: the query is a vector
  - We regard the query as a short document
  - We return the documents ranked by the closeness of their vectors to the query (also a vector)

$$sim(q, d_i) = \frac{\vec{V}(q) \cdot \vec{V}(d_i)}{|\vec{V}(q)||\vec{V}(d_i)|}$$

  - Note that q is very sparse!

Cosine Similarity Score

$$
\begin{aligned}
\vec{V}(d_1) \cdot \vec{V}(d_2) &= cos(\theta) \cdot |\vec{V}(d_1)||\vec{V}(d_2)| \\
cos(\theta) &= \frac{\vec{V}(d_1) \cdot \vec{V}(d_2)}{|\vec{V}(d_1)||\vec{V}(d_2)|} \\
sim(d_1, d_2) &= \frac{\vec{V}(d_1) \cdot \vec{V}(d_2)}{|\vec{V}(d_1)||\vec{V}(d_2)|}
\end{aligned}
$$



Cosine Similarity Score

- Define: dot product

$$\vec{V}(d_1) \cdot \vec{V}(d_2) = \sum_{i=t_1}^{t_n} (\vec{V}(d_1)_i \vec{V}(d_2)_i)$$

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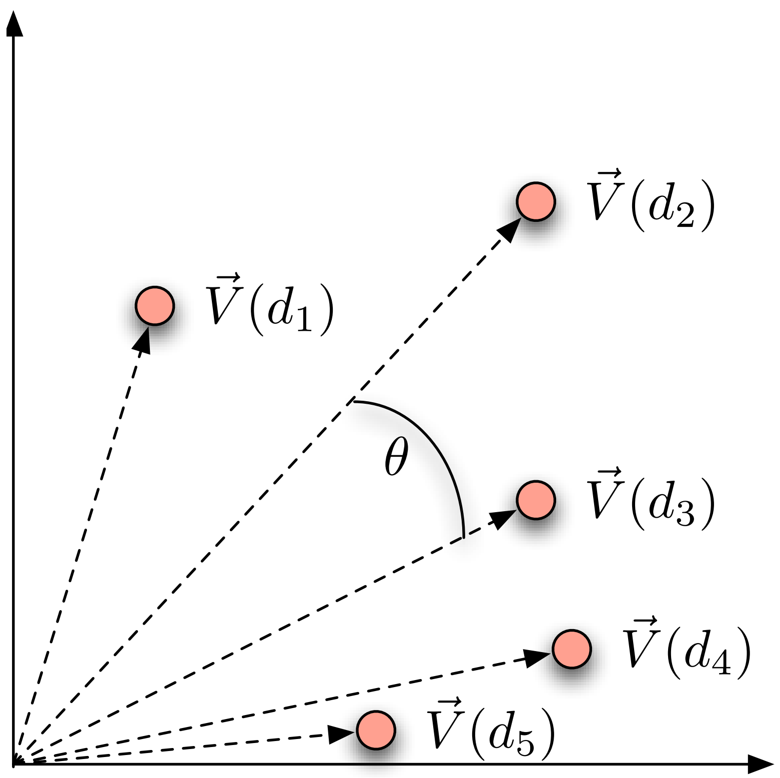

$$\begin{aligned}\vec{V}(d_1) \cdot \vec{V}(d_2) &= (13.1 * 11.4) + (3.0 * 8.3) + (2.3 * 2.3) + (0 * 11.2) + (17.7 * 0) + (0.5 * 0) + (1.2 * 0) \\ &= 179.53\end{aligned}$$

Cosine Similarity Score

- Define: Euclidean Length

$$|\vec{V}(d_1)| = \sqrt{\sum_{i=t_1}^{t_n} (\vec{V}(d_1)_i \vec{V}(d_1)_i)}$$

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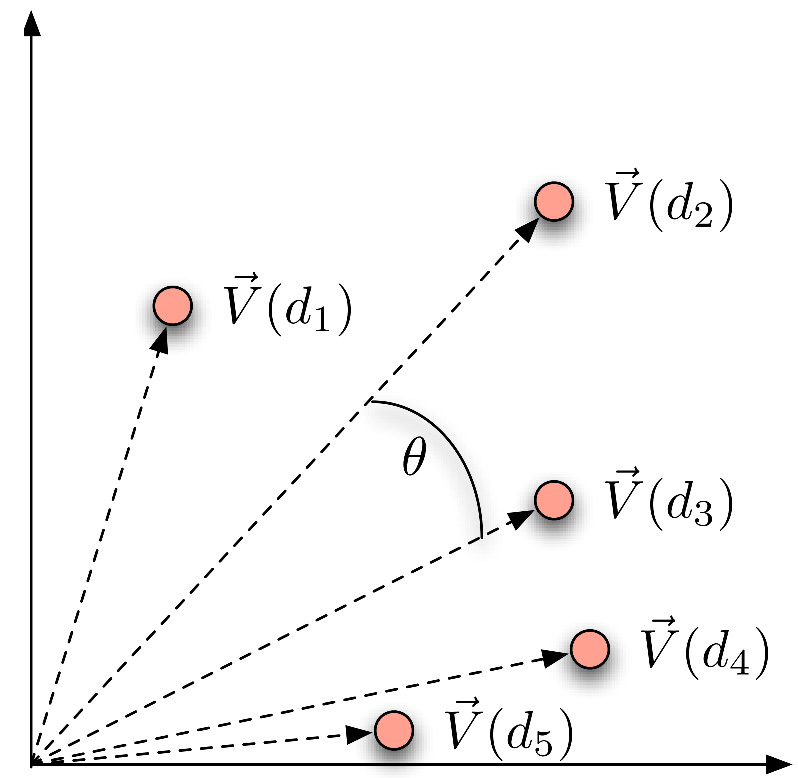

$$\begin{aligned} |\vec{V}(d_1)| &= \sqrt{(13.1 * 13.1) + (3.0 * 3.0) + (2.3 * 2.3) + (17.7 * 17.7) + (0.5 * 0.5) + (1.2 * 1.2)} \\ &= 22.38 \end{aligned}$$

Cosine Similarity Score

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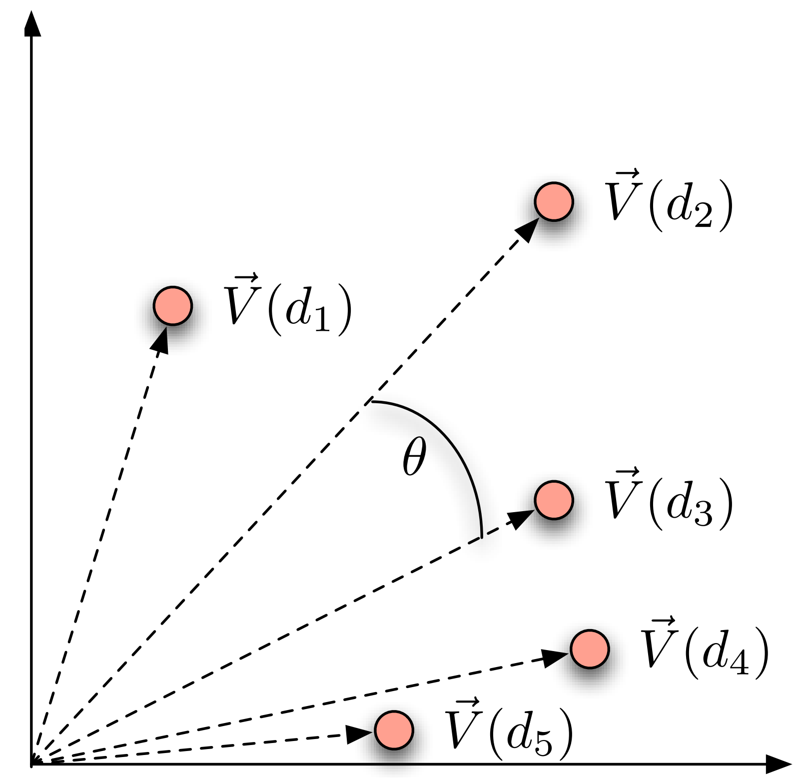
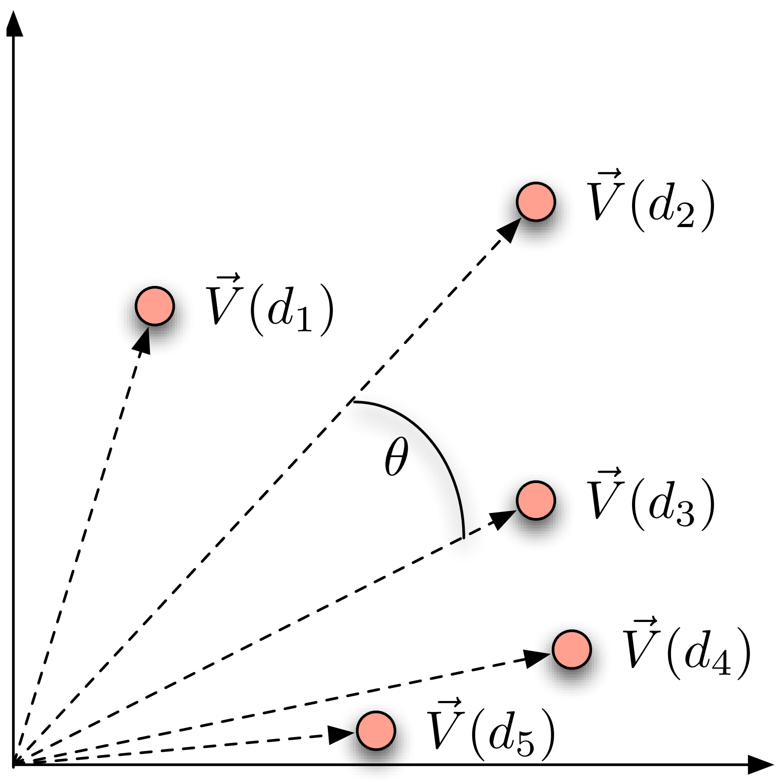


$$|\vec{V}(d_1)| = \sqrt{(11.4 * 11.4) + (8.3 * 8.3) + (2.3 * 2.3) + (11.2 * 11.2)}$$
$$= 18.15$$

Cosine Similarity Score

- Example

$$
\begin{aligned}
sim(d_1, d_2) &= \frac{\vec{V}(d_1) \cdot \vec{V}(d_2)}{|\vec{V}(d_1)||\vec{V}(d_2)|} \\
&= \frac{179.53}{22.38 * 18.15} \\
&= 0.442
\end{aligned}
$$



Exercise

- Rank the following by decreasing cosine similarity.
  - Assume tf-idf weighting:
    - Two docs that have only frequent words in common
      - (the, a , an, of)
    - Two docs that have no words in common
    - Two docs that have many rare words in common
      - (mocha, volatile, organic, shade-grown)

Spamming indices

- This was invented before spam
- Consider:
  - Indexing a sensible passive document collection
  - vs.
  - Indexing an active document collection, where people, companies, bots are shaping documents to maximize scores
- Vector space scoring may not be as useful in this context.

Interaction: vectors and phrases

- Scoring phrases doesn’t naturally fit into the vector space world:
  - How do we get beyond the “bag of words”?
  - “dark roast” and “pot roast”
  - There is no information on “dark roast” as a phrase in our indices.
- Biword index can treat some phrases as terms
  - postings for phrases
  - document wide statistics for phrases

Interaction: vectors and phrases

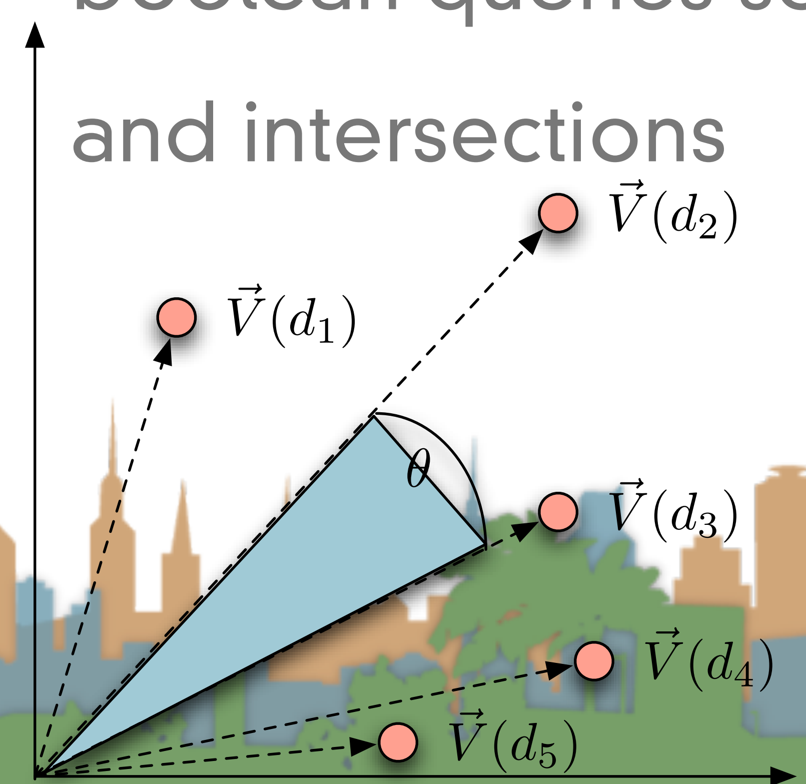
- Theoretical problem:
  - Axes of our term space are now correlated
    - There is a lot of shared information in “light roast” and “dark roast” rows of our index
- End-user problem:
  - A user doesn’t know which phrases are indexed and can’t effectively discriminate results.

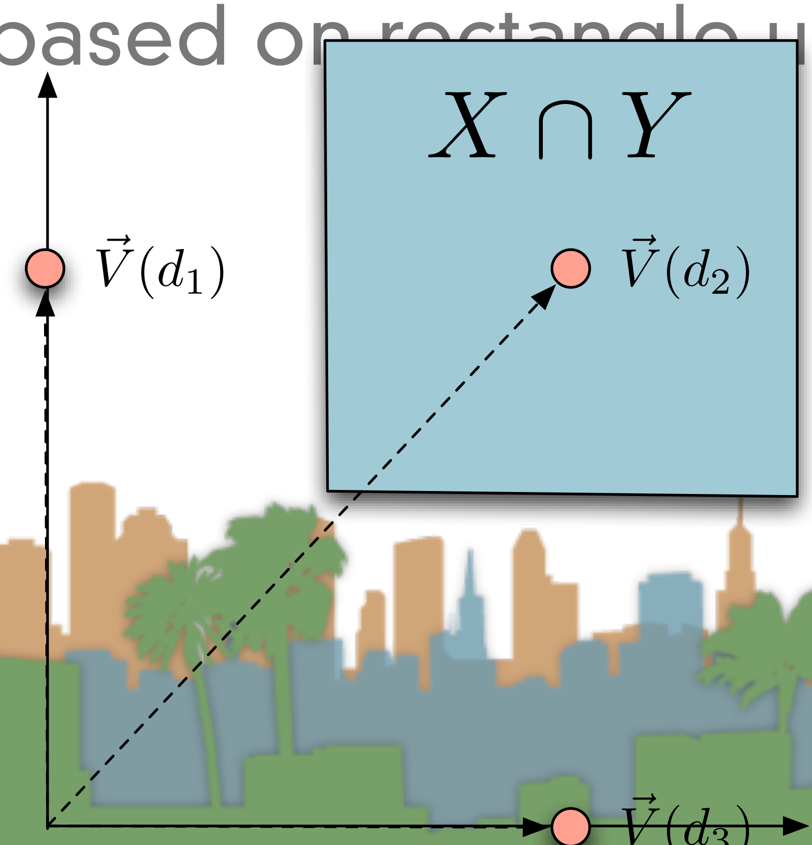
Multiple queries for phrases and vectors

- Query: “rising interest rates”
- Iterative refinement:
  - Run the phrase query vector with 3 words as a term.
  - If not enough results, run 2-phrase queries and fold into results: “rising interest” “interest rates”
  - If still not enough results run query with three words as separate terms.

Vectors and Boolean queries

- Ranked queries and Boolean queries don't work very well together
    - In term space
        - ranked queries select based on sector containment - cosine similarity
        - boolean queries select based on rectangle unions and intersections





Vectors and wild cards

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Vectors and wild cards

- How could we work with the query, “quick* print*” ?
  - Can we view this as a bag of words?
  - What about expanding each wild-card into the matching set of dictionary terms?
- Danger: Unlike the boolean case, we now have tfs and idfs to deal with
- Overall, not a great idea

Vectors and other operators

- Vector space queries are good for no-syntax, bag-of-words queries
  - Nice mathematical formalism
  - Clear metaphor for similar document queries
  - Doesn’t work well with Boolean, wild-card or positional query operators
  - But ...

Query language vs. Scoring

- Interfaces to the rescue
  - Free text queries are often separated from operator query language
  - Default is free text query
  - Advanced query operators are available in “advanced query” section of interface
  - Or embedded in free text query with special syntax
    - aka -term -”terma termb”

Alternatives to tf-idf

- Sublinear tf scaling
    - 20 occurrences of "mole" does not indicate 20 times the relevance
    - This motivated the WTF score.

$$
\begin{array}{lll}
\multicolumn{3}{l}{\mathrm{WTF}(t, d)} \\
1 & \textbf{if } tf_{t,d} = 0 & \\
2 & \quad \textbf{then } return(0) & \\
3 & \quad \textbf{else } \; return(1 + log(tf_{t,d})) &
\end{array}
$$

- There are other variants for reducing the impact of repeated terms

LUCI