
Exploring UFO's

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CONTENTS

i. Glossary	2
ii. Abbreviations	3
iii. Notation	4
1 Background	5
2 UFO	5
3 Properties	7
4 Preliminary Tests of AOBB-UFO on UAI 2022 Competition MMAP	9

GLOSSARY

-underflowed function

(f) A function such that output values less than ϵ are replaced with 0.0.

$$f(a) = \begin{cases} f(a); & f(a) \geq \epsilon \\ 0; & \text{otherwise:} \end{cases}$$

4

graphical model

($M = \langle \mathcal{X}; \mathcal{D}; F \rangle$). Mathematical tool for modeling complex systems composed of a set of variables \mathcal{X} , a set of domains $\mathcal{D} = \{D_X \mid X \in \mathcal{X}\}$ for each variable X , and a set of functions F with each function defined over a subset of the model's variables \mathcal{X} . 4

marginal maximum a-posteriori

(MMAP). The marginal likelihood associated with the configuration of a target subset of variables Q that maximizes their marginal likelihood.

In the context of discrete graphical models without evidence, with $Q \subseteq \mathcal{X}$, $S = \mathcal{X} \setminus Q$ be the variables to sum over, $F_Q = \{f \mid \text{scope}(f) \subseteq Q\}$ be the set of functions defined only over Q , and $F_S = F \setminus F_Q$ be functions that include some $X \in S$ in their scope,

$$MMAP = \max_Q \sum_{S} \prod_{f \in F} f(q, s) \quad (1)$$

$$= \max_Q \sum_{f \in F_S} \prod_{f \in F_S} f^0(q) \quad (2)$$

3, 4, 7

maximum a-posteriori

(MAP). With respect to a graphical model, the likelihood value associated with the **most probable explanation** or **MPE**.

In the context of discrete graphical models without evidence, $MAP = \max_{Q \subseteq \mathcal{X}} \prod_{f \in F} f(q)$. 3, 4

most probable explanation

(MPE). Assignment to variables the variables of a graphical model that maximizes the conditional probability of the observed evidence.

In the context of discrete graphical models without any evidence, $MPE = \arg\max_{Q \subseteq \mathcal{X}} \prod_{f \in F} f(q)$. 2, 3

partial configuration

A joint assignment to a subset of the variables of a graphical model. 4

partition function

(Z). A mathematical quantity that characterizes the distribution among a system's possible states and serves as a normalizing constant for calculating probabilistic measures associated with these states.

In the context of discrete graphical models, $Z = \sum_{\mathbf{x}} \prod_{f \in F} f(\mathbf{x})$. 3

ABBREVIATIONS

MAP

Maximum a-posteriori. 7

MMAP

Marginal maximum a-posteriori. 7

MPE

Most probable explanation. 2

Z

Partition function. 7

NOTATION

[capital letters] (ex. X)

Represents a variable of a **graphical model**. 7

[lower-case letters] (ex. x)

Represent assignments to variable corresponding to their capitalized form. For example, x represents a particular assignment to the variable represented by X , or $X = x$. 7

[bold-faced capital letters] (ex. \mathcal{X})

A set of variables of a **graphical model**. (\mathcal{X} , in particular, often refers to the set of all variables of a graphical model). 7

X

The set of all variables X of a **graphical model**. 4

Q

In the context of the **maximum a-posteriori** or **marginal maximum a-posteriori** task, the [sub]set of the variables Q that are to be maximized over (know as "query" or "MAP" variables). 7

S

In the context of the **marginal maximum a-posteriori** task, the subset of variables to be summed over. $S = \mathcal{X} \setminus Q$ 7

x

A full configuration, ie. assignment to all variables \mathcal{X} of a **graphical model**.

$X = x \in D_X$. 7

D_Y

The set of all possible **partial configurations** to the variables of the subset $Y \subseteq \mathcal{X}$. D_Y is the Cartesian product of the domains of the variables in Y . $D_Y = \prod_{Y \ni x} D_x$, where \prod is the Cartesian product operator. 7

F

$F = \{f \mid f \in \mathcal{F}\}$ 7

F_Q

In the context of the **maximum a-posteriori** or **marginal maximum a-posteriori** task, the subset of functions defined only on a subset of the variables Q that are to be maximized over (know as "query" or "MAP" variables).

$F_Q = \{f \mid f \in \mathcal{F}_Q\}$ 7

F_S

In the context of the **marginal maximum a-posteriori** task, $F_S = F \setminus F_Q = \{f \mid f \in \mathcal{F}, \text{ s.t. } X \setminus Q \subseteq \text{dom}(f)\}$ 7

\mathcal{M}

Graphical model $\mathcal{M} = \langle \mathcal{X}; D; F \rangle$ with non-negative functions. 7

$\tilde{\mathcal{M}}$

An altered **Graphical model** with non-negative functions such that each original function is replaced by its corresponding **-underflowed function**. $\tilde{\mathcal{M}} = \langle \mathcal{X}; D; \tilde{F} \rangle$. 7

$\text{MMAP}(\mathcal{M}; Q)$

The **marginal maximum a-posteriori** of graphical model \mathcal{M} maximizing over the subset of variables Q . 7

1 BACKGROUND

Probabilistic graphical models are powerful tools for modeling complex systems with local structure. A discrete graphical model can be defined as a 3-tuple $\mathcal{M} = \langle \mathbf{X}; \mathbf{D}; \mathbf{F} \rangle$, where: \mathbf{X} is a set of variables for which the model is defined; $\mathbf{D} = \{D_X : X \in \mathbf{X}\}$ is a set of finite domains, each defining the possible assignments for a variable; each $f \in \mathbf{F}$ is a real-valued function defined over a subset of the model's variables \mathbf{X} known as the function's **scope**. More concretely, if we let D denote the Cartesian product of the domains of the variables in \mathbf{X} , then $f : D \rightarrow \mathbb{R}_0$. We let capital letters (X) represent variables and small letters (x) represent their assignment. Boldfaced capital letters (\mathbf{X}) denote a collection of variables, $|\mathbf{X}|$ its cardinality, $D_{\mathbf{X}}$ its joint domain, and \mathbf{x} a particular realization in that joint domain called a **configuration**. Operations denoted $\sum_{\mathbf{x}}$ (ex. $\sum_{x \in \mathbf{X}}$) imply $\sum_{\mathbf{x} \in D_{\mathbf{X}}}$.

Some common queries to a graphical model include asking for the most likely assignment to the variables, marginal probabilities of some variables, or more complex tasks such as determination of the **marginal maximum a-posteriori (or MMAP)** (Definition 1.0.0.1, [Dechter, 2019]).

Definition 1.0.0.1 (MMAP)

Given a graphical model $\mathcal{M} = \langle \mathbf{X}; \mathbf{D}; \mathbf{F} \rangle$, the marginal maximum a-posteriori of \mathcal{M} is:

$$\text{MMAP}(\mathcal{M}; \mathbf{Q}, \mathbf{X}) = \max_{\mathbf{Q}} \sum_{\mathbf{S} = \mathbf{X} \cup \mathbf{Q}} f(\mathbf{q}; \mathbf{s}) \quad (3)$$

Answering these queries involve computational tasks that are NP-Hard thus for large models obtaining reasonable approximations is often the goal. To this end, many approximate algorithms have been developed for a wide variety of these tasks. In this work, we add a new tool to this arsenal in the form of a new algorithm called UFO.

2 UFO

Utilizing constraint propagation (CP) as a tool for pruning inconsistent search paths has been shown to be able to greatly speed up search [Dechter [2019], Mateescu and Dechter [2008], Darwiche [2009]]. Similar ideas have been explored in mixed integer programming [Danna et al., 2005]. More recently in the scope of protein design Pezeshki et al. [2022] demonstrated that introducing artificially generated determinism by underflowing function values under a provided threshold (Definition 2.0.0.1) can further leverage CP and enhance the speed of solving K^* optimization problems. Thus, being able to identify a good threshold and use it to speed up search could be invaluable to solving certain problems.

Definition 2.0.0.1 (ϵ -underflow of f, F)

Let f be a non-negative function and $\epsilon \in \mathbb{R}^+$. The ϵ -underflow of f is $f^\epsilon(x) = f(x)$ if $f(x) \geq \epsilon$ and 0, otherwise.

Definition 2.0.0.2 (ϵ -underflow of \mathcal{M}, \mathcal{M})

For $\mathcal{M} = \langle \mathbf{X}; \mathbf{D}; \mathbf{F} \rangle$, the ϵ -underflow of \mathcal{M} is $\mathcal{M}^\epsilon = \langle \mathbf{X}; \mathbf{D}; \mathbf{F}^\epsilon \rangle$, where $\mathbf{F}^\epsilon = \{f^\epsilon : f \in \mathbf{F}\}$.

Definition 2.0.0.3 (Inconsistent Model)

A model is said to be inconsistent if $\sum_{\mathbf{x} \in D_{\mathbf{X}}} f(\mathbf{x}) = 0$.

Choosing a larger underflow-threshold leads to more underflows and consequently more infused determinism thus resulting in more aggressive pruning via CP. However, if the threshold is set too high, the resulting model becomes inaccurate and may even become inconsistent altogether, resulting in pruning of all configurations. Therefore it is useful to find a threshold that is as high as possible yet still results in a consistent model.

Algorithm 1: UFO (underflow-threshold optimization) describes a general methodology for choosing such an underflow-threshold. To achieve this UFO employs binary search to find the largest threshold that still results in a satisfiable model (lines 6-12). Then UFO decreases the threshold using a hyper-parameter δ (line 14) to enable a wider array of solutions.

Note that UFO operates under the assumption that satisfiability of a model can be determined quickly. This is not true in general, nevertheless we have found that the satisfiability sub-task underlying many optimization problems tends to be easy. In other cases, satisfiability can be approximated by constraint propagation schemes [Dechter, 2003].

Algorithm 1: UFO

input : Graphical model $\mathcal{M} = \langle \mathbf{X}, \mathbf{D}, \mathbf{F} \rangle$; SAT solving algorithm, $SAT(\cdot)$; time limit for binary search; a deflation factor $0 < \delta \leq 1$

output : A proposed threshold τ to use

```
1 begin
2   if  $SAT(\mathcal{M}) = False$  then
3     | return FAILURE
4    $\tau_{min} = 0$ ;  $\tau_{max} = \max_{\mathbf{F}, \mathbf{X}} f(\mathbf{x})$ 
5    $\tau = \frac{\tau_{max} + \tau_{min}}{2}$ 
6   while time remains for  $\tau$  binary search do
7     | if  $SAT(\mathcal{M}_\tau) = False$  then
8       |  $\tau_{max} = \tau$ 
9     | else
10      |  $\tau_{min} = \tau$ 
11      |  $\tau = \frac{\tau_{max} + \tau_{min}}{2}$ 
12    end
13     $\tau = \tau_{min} \cdot \delta$ 
14  return  $\tau$ 
15 end
```

3 PROPERTIES

Theorem 3.0.0.1 (Lower-bounds from ϵ -underflows)

$\delta \text{task} \geq fZ; \text{MAP}; \text{MMAP}g; \delta M; \text{task}(M) \geq \text{task}(M)$

Proof

The evaluated value of summation, maximization, or mixed max-sum operations over products of non-negative functions can only decrease if function values decrease (but remain non-negative). By definition, ϵ -underflows either do not affect function values or decrease them to 0.0. Thus, the evaluated value of summation, maximization, or mixed max-sum operations can only decrease, and never increase, due to a ϵ -underflows. \square

Corollary 3.0.0.2 (Monotonicity from ϵ -underflows)

$\delta \text{task} \geq fZ; \text{MAP}; \text{MMAP}g; \delta M; \delta^0 < \epsilon; \text{task}(M) \geq \text{task}(M)$.

Proof

$\delta M; \delta^0 < \epsilon; M = M$. Thus, the result follows directly from Theorem 3.0.0.1 \square

Corollary 3.0.0.3 (Solution persistence)

$\delta \text{task} \geq fZ; \text{MAP}; \text{MMAP}g$, if a ϵ -underflow does not affect the function values involved in the computation of the solution cost, then $\text{task}(M) = \text{task}(M)$.

More formally, $\delta M; \text{if } \delta f \geq F; f = f; \text{ then } Z(M) = Z(M) \text{ and } \delta \text{task} \geq f\text{MAP}; \text{MMAP}g; \delta M; \text{if } \exists Q$
 $q \in \text{task}(M) \text{ then } \text{task}(M) = \text{task}(M); \delta f \geq F; f(q) = f(q); \text{ then } \text{task}(M) = \text{task}(M)$.

Proof

Case 1 (Z):

If $\delta f \geq F; f = f$; then $M = M$ and $\text{task}(M) = \text{task}(M)$ follows trivially.

Case 2 (MAP and MMAP):

If $\exists Q \quad q \in \text{task}(M) \text{ then } \text{task}(M) = \text{task}(M); \delta f \geq F; f(q) = f(q); \text{ then } \text{task}(M) = \text{task}(M)$. Since $\delta q \notin Q; \text{task}(M) = \text{task}(M)$ and by ϵ -underflows can only decrease costs for these tasks, the optimal solution in the ϵ -underflowed problem will still correspond to q . \square

Theorem 3.0.0.4 (Tractable upper-bound on error $\text{MMAP}(M; Q) - \text{MMAP}(M_S; Q)$)

Given a graphical model M for computing the **marginal maximum a-posteriori** maximizing over query variables $Q \subseteq X$ and a ϵ -underflow of the model, M_S , such that underflows are only applied to functions F_S that include summation variables in their scope ($F_S = \prod_{j \in S} f_j$ and $\exists X \subseteq S \text{ s.t. } X \subseteq S = X \cap Q$), the error $\text{MMAP}(M; Q) - \text{MMAP}(M_S; Q)$ is upper-bounded by

$$\sum_{Q \subseteq X} \prod_{j \in Q} (v_{F_j})^{f_j} \prod_{j \in S} (v_{F_j})^{f_j} \quad (1)$$

Proof

Let $S = X \cap Q$ be the set of variables to sum over, $F_Q = \prod_{j \in Q} f_j$ be the set of functions defined only over Q , and $F_S = \prod_{j \in S} f_j$ be functions that include some $X \subseteq S$ in their scope, the MMAP can be expressed as:

$$\text{MMAP}(M; Q) = \max_Q \prod_{j \in Q} f_j(q) \prod_{j \in S} f_j(q) \quad (4)$$

for which the summation can be more explicitly be expressed via the notation

$$\text{MMAP}(M; Q) = \max_Q \prod_{j \in Q} f_j(q) \prod_{j \in S} f_j(q) \quad (5)$$

Assume the MMAP solution is due to some unique assignment $Q = q$. (Although a MMAP solution can be due to a set of possible q 's, it will be easy to see that the proof pertaining to a unique MMAP q assignment is easily extendable to cases with multiple assignments). The MMAP value can now be expressed as

$$\text{MMAP}(M; Q) = Z(M; q) = \prod_{j \in Q} f_j(q) \prod_{j \in S} f_j(q) \quad (6)$$

Now partition D_S into subsets $D_S^{(q)}$ and $D_{S^c}^{(q)}$ where

$$D_S^{(q)} = \{s \in D_S; f(q, s) \geq g\} \quad (7)$$

$$D_{S^c}^{(q)} = \{s \in D_S; f(q, s) < g\} \quad (8)$$

$D_S^{(q)}$ corresponds to terms in the summation that are not affected by a δ -underflow of the problem whereas $D_{S^c}^{(q)}$ corresponds to terms of the summation that end up being 0.0 after a δ -underflow of the model is performed. As such, the cost of q in the δ -underflowed model is

$$Z(M_S; q) = \sum_{s \in D_S} f^0(q, s) \quad (9)$$

$$= \sum_{s \in D_S} f^0(q, s) \left(\sum_{s \in D_S} f^{00}(q, s) + \sum_{s \in D_{S^c}^{(q)}} f^{00}(q, s) \right) \quad (10)$$

$$= \sum_{s \in D_S} f^0(q, s) \sum_{s \in D_S} f^{00}(q, s) + \sum_{s \in D_{S^c}^{(q)}} f^0(q, s) \sum_{s \in D_{S^c}^{(q)}} f^{00}(q, s) \quad (11)$$

$$= \sum_{s \in D_S} f^0(q, s) \sum_{s \in D_S} f^{00}(q, s) + 0.0 \quad (12)$$

$$= \sum_{s \in D_S} f^0(q, s) \sum_{s \in D_S} f^{00}(q, s) \quad (13)$$

Thus, the error $= MMAP(M; Q) - MMAP(M_S; Q)$ can be expressed as

$$= \sum_{s \in D_S} f^0(q, s) \sum_{s \in D_{S^c}^{(q)}} f^{00}(q, s) \quad (14)$$

and, since for each $s \in D_S$ at least one $f^{00}(q, s)$ has to evaluate to be zero by definition of D_S , can be at most

$$(v_{F_Q})^{j_{F_Q}} \sum_{s \in D_{S^c}^{(q)}} (v_{F_S})^{j_{F_S} - 1} \quad (15)$$

□

Complexity

Testing the condition in Corollary 3.0.0.4 is can be done in linear time.

4 PRELIMINARY TESTS OF AOBB-UFO ON UAI 2022 COMPETITION MMAP

Tables 1- 2 shows preliminary test results of AOBB [Marinescu et al., 2018] empowered with UFO. For these preliminary results, the UFO binary search was done over log space for two seconds and no deflation was applied to the resulting threshold. Per the competition restrictions, the algorithm was constrained to using 8G of memory.

Table Key

For each problem, provided are:

- $|X| ; |F|$: the number of variables and functions, respectively
- w^* : the induced width (see Extended Supplemental) due to the variable ordering used by AOBB-UFO
- k : the maximum domain size
- *Anytime*: the time it took AOBB-UFO to originally find its final solution
- *Time*: the time it took AOBB-UFO to terminate
- *Solution*: AOBB-UFO's best found solution
- the best found solution for each of the competing solvers

Problem	PROBLEM STATISTICS				AOBB-UFO			UAI 2022 COMPETITION COMPETING SOLVERS							
	$ X ; F $	w^*	k		Anytime	Time	Solution	braobb	daoopt-lh	daoopt	lbp	toulbar2			uai14
												ipr	vacint	vns	
75-17-5.Q0.5.14	289 ; 289	110	2		18.1	18.06	-7.7	-7.5	-7.7	-7.7		-7.7	-7.7	-7.7	-14.1
75-19-5.Q0.5.12	361 ; 361	133	2		15.7	15.66	-10.1	-9.7	-9.6	-9.6		-9.6	-9.6	-9.6	-15.9
75-22-5.Q0.5.12	484 ; 484	110	2		37.6	37.59	-12.8		-11.8	-11.7		-11.7	-11.7	-11.7	
75-23-5.Q0.5.13	529 ; 529	177	2		17.9	17.95	-13.7	-13.8	-12.5	-12.5		-12.5	-12.5	-12.5	
75-26-5.Q0.5.14	676 ; 676	208	2		31.5	31.46	-19	-18.3	-18.1	-18.1		-18.1	-18.1	-18.1	
90-22-5.Q0.5.14	484 ; 484	173	2		39.4	39.39	-6.4	-5.6	-5.6	-5.6		-5.6	-5.6	-5.6	
90-24-5.Q0.5.12	576 ; 576	131	2		27.4	27.37	-5.8	-5.6	-5.7	-5.7		-5.7	-5.7	-5.7	
90-25-5.Q0.5.12	625 ; 625	174	2		21.4	21.44	-7.9		-7.8	-7.7		-7.7	-7.7	-7.7	
90-26-5.Q0.5.11	676 ; 676	110	2		27	26.96	-8.8		-9.1	-8.7		-8.7	-8.7	-8.7	
90-30-5.Q0.5.11	900 ; 900	246	2		36.1	36.14	-11	-11.5	-10.9	-10.9		-10.9	-10.9	-10.9	
90-34-5.Q0.5.12	1156 ; 1156	352	2		29.6	29.63	-12.7		-14.2	-12.2		-12.2	-12.2	-12.2	
90-38-5.Q0.5.14	1444 ; 1444	371	2		44.9	44.86	-17.1			-16.8		-16.8	-16.8	-16.8	
90-42-5.Q0.5.14	1764 ; 1764	300	2		40.6	40.59	-17.3			-17		-17	-17	-17	
90-46-5.Q0.5.14	2116 ; 2116	356	2		46	45.95	-26			-24.8		-24.9	-24.9	-24.5	
90-50-5.Q0.5.13	2500 ; 2500	788	2		201.2	201.21	-26.3	-36.9		-26.8		-25.7	-25.7	-25.7	
bw_p24_16	937 ; 937	136	3			82.8	3682.8								
bw_p24_20	1169 ; 1169	171	3		105.2	3705.2									
bw_p34_15	2191 ; 2191	294	3		59.2	3659.2									
bw_p34_20	2916 ; 2916	389	3		108.5	3708.5									
bw_p44_15	4075 ; 4075	638	3		514.1	4114.1									
bw_p44_19	5155 ; 5155	722	3		828.9	4428.9									
bw_p54_10	4366 ; 4366	777	3		911.2	911.25	-1								
bw_p54_16	6964 ; 6964	1134	3												
comm_p01_16	4477 ; 4477	831	2		266.9	3866.9									
comm_p01_20	5585 ; 5585	1039	2		453.8	4053.8									
Grids_20	6400 ; 19040	122	2		63.8	3663.8		4518.7	4833.9	4839	4804.9	4837.5	4816.7	4836.5	
Grids_21	1600 ; 4800	108	2		50.9	3650.9		8002.4	8429.3	8499.7	8322.9	8499.7	8357.8	8438.1	8222.4
Grids_22	1600 ; 4800	97	2		53.1	3653.1		2687.2	2833.8	2835.2	2763.4	2835.2	2797.1	2823.4	2790.8
Grids_23	1600 ; 4720	73	2		40	3640		2787.3	2793.4	2793	2739.2	2793	2778.7	2780.3	2765
Grids_24	1600 ; 4720	61	2		40.9	3640.9		8204.5	8234.4	8237.3	8106.5	8237.3	8160.9	8175.2	8023.4
Grids_25	1600 ; 4720	63	2		71.2	3671.2		1209.9	1208.9	1209.3	1204.6	1209.3	1199.5	1209.1	1209
Grids_26	400 ; 1200	59	2		58.7	3658.7		1322.2	1326.3	1326.3	1300.4	1326.3	1321.4	1322.2	1280.9
Grids_27	1600 ; 4720	69	2		38.5	3638.5		5507	5506.8	5508.9	5383.1	5508.9	5422.5	5489.7	5378.4
Grids_28	400 ; 1200	53	2		90	3690		1969.1	1982.5	1982.5	1924.7	1982.5	1978.2	1972.6	1948.2
Grids_29	400 ; 1200	54	2		83.6	3683.6		669.2	673	673	662.6	673	672.3	670.4	666.3

Table 1: AOBB-UFO on UAI 2022 Competition Final Problems (3600s)

Problem	PROBLEM STATISTICS				AOBB-UFO			UAI 2022 COMPETITION COMPETING SOLVERS							
	X ; F	w*	k	Anytime	Time	Solution	braobb	daoopt-lh	daoopt	lbp	toulbar2			uai14	
											ipr	vacint	vns		
ImageAlignment_11	350 ; 3563	33	77	25.5	108.67	-1005.8	-824.2	-824.2	-824.2	-824.2	-824.2	-824.2	-824.2	-824.2	
ImageAlignment_12	30 ; 465	29	58	5.3	5.28	-436.7	-436.7	-436.7	-436.7	-436.7	-436.7	-436.7	-436.7	-436.7	
ImageAlignment_13	400 ; 3334	21	83	78	128.24	-3081.4	-2998.9	-2998.8	-2998.8	-2998.8	-2998.8	-2998.8	-2998.8	-2998.8	
ImageAlignment_14	200 ; 2128	23	69	9.1	12.89	-1632	-1557.5	-1557.5	-1557.5	-1557.5	-1557.5	-1557.5	-1557.5	-1557.5	
ImageAlignment_15	300 ; 2732	23	68	59.7	415.05	-1339.5	-1177.5	-1177.5	-1177.5	-1177.5	-1177.5	-1177.5	-1177.5	-1177.5	
ObjectDetection_13	60 ; 1830	59	21	6.2	3606.1	-528.6	6684.3	9967.7	9854.2	8826.5	9970.6	9970.6	9970.6	8883.1	
ObjectDetection_14	60 ; 1830	59	11	31.6	3631.1	236.6	6712.5	9020.8	9020.8	8341.9	9093.7	9093.7	9093.7	8562.9	
ObjectDetection_15	60 ; 1830	59	16	2155.8	3637	-404	10628	11703.8	12479	11544	12633	12633	12633	11976	
ObjectDetection_16	60 ; 1830	59	21	1753.5	3606.2	-471	12023	14154.2	13536	13548	14347	14347	14347	14022	
ObjectDetection_17	60 ; 1830	59	11	30.6	3630.5	-335.9	2454.9	4716.4	4816	3794.9	4887.4	4887.4	4887.4	4518.2	
or_chain_11.fg.Q0.5.I3	900 ; 915	191	2	30.8	30.85	-22.9									
or_chain_16.fg.Q0.5.I3	1675 ; 1700	318	2	98.5	98.52	-38.1	-53.8	-25.2	-23.4		-23.4	-23.4	-23.4		
or_chain_22.fg.Q0.5.I3	1044 ; 1054	196	2	28.9	28.92	-15.3									
or_chain_24.fg.Q0.5.I3	1155 ; 1171	247	2	32.3	32.31	-24.4									
or_chain_25.fg.Q0.5.I3	1075 ; 1086	88	2	30.4	30.37	-16.8									
or_chain_32.fg.Q0.5.I3	1466 ; 1478	108	2	32	3632										
or_chain_36.fg.Q0.5.I3	933 ; 943	91	2	30.5	30.46	-15.3									
or_chain_39.fg.Q0.5.I3	1751 ; 1766	430	2	96.3	96.32	-22.9									
or_chain_40.fg.Q0.5.I3	988 ; 998	96	2	16.3	16.26	-15.1									
or_chain_41.fg.Q0.5.I3	1847 ; 1863	203	2	118.5	3645	-27									
or_chain_43.fg.Q0.5.I3	1692 ; 1712	216	2	44.5	44.5	-30.5									
or_chain_6.fg.Q0.5.I3	1849 ; 1876	386	2	2351.2	3710.9	-41.2									
or_chain_60.fg.Q0.5.I3	1997 ; 2023	552	2	3691.2	3736.6	-42.8									
or_chain_63.fg.Q0.5.I3	731 ; 744	97	2	26.3	26.35	-10.8									
or_chain_8.fg.Q0.5.I3	1195 ; 1203	63	2	23.6	23.58	-12.2									
pedigree1.Q0.5.I3	298 ; 334	105	4	3319.4	3641.4	-35.2		-35.9	-37.3		-35.7	-35.7	-36.1	-37.8	
pedigree13.Q0.5.I1	888 ; 1077	138	3	2941.6	3623.5	-63.1		-62.8	-62.4		-62.8	-62.7	-62.5		
pedigree18.Q0.5.I1	931 ; 1184	181	5	38.8	3638.8			-112.5	-111.9		-112.5	-112.9	-113.5	-116.4	
pedigree19.Q0.5.I4	693 ; 793	173	5	3280	3625.2	-97.5	-95.3	-89.5	-87.5		-87.1	-89.5	-89.6		
pedigree20.Q0.5.I2	387 ; 437	76	5	2455.1	3614.7			-46.8	-46.7		-47.9	-46.1	-46.1	-72.8	
pedigree25.Q0.5.I2	993 ; 1289	178	5	3236.3	3702.4	-148.7	-154.6	-148.5	-147.8		-147.6	-147.3	-147.8	-170	
pedigree30.Q0.5.I2	1015 ; 1289	109	5	30.9	3630.9			-123.6	-124.7		-123.6	-123.1	-124.3	-140.8	
pedigree31.Q0.5.I2	1006 ; 1183	153	5	16	3616		-118.4	-116.2	-116		-117	-117.6	-115.6	-161.1	
pedigree33.Q0.5.I2	581 ; 798	118	4	28.7	3628.7		-69.4	-70.3	-70.9		-71	-70.4	-71.1		
pedigree38.Q0.5.I2	581 ; 724	164	5	115.5	3621.9	-94.2	-91.4	-78.2	-77.6		-77.5	-77.6	-77.8	-90.5	
pedigree41.Q0.5.I2	885 ; 1062	205	5	27.5	3620.7	-117		-105.6	-107.3		-106.8	-107.7	-108.2	-160.7	
pedigree44.Q0.5.I4	644 ; 811	186	4	3177.3	3628.3	-89.3	-98.2	-89.1	-88.5		-89.1	-89.4	-89.7		
pedigree50.Q0.5.I1	478 ; 514	124	6	214.5	3677.4	-51.5	-55.7	-52.4	-52.8		-53.1	-53.1	-52	-69.6	
pedigree7.Q0.5.I2	867 ; 1068	89	4	3579.1	3628.3	-103.8		-97.5	-97.6		-98.4	-98	-98.3	-136.4	
pedigree9.Q0.5.I3	935 ; 1118	235	7	913.3	3626.6	-123.9		-113.8	-113.7		-113.4	-112.7	-114	-159	
pomdp10-12_7_3_8_4	2673 ; 2701	2599	32				1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	
pomdp6-12_6_2_6_3	250 ; 265	198	18	63.9	3663.9	0.9	1	1	1	1	1	1	1	1	
pomdp7-20_10_2_10_3	3166 ; 3193														
pomdp8-14_9_3_12_4	2145 ; 2189	2057	48				1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	
pomdp9-14_8_3_10_4	5277 ; 5313														
ProteinFolding_11	400 ; 1160	28	2	22.4	22.44	1926.6	1962.3	1962.3	1962.3	1839.4	1962.3	1959.7	1944.8	1528.7	
ProteinFolding_12	250 ; 2098	20	60	10	13.34	-1586.4	-1547	-1547	-1547	-1548	-1547	-1547	-1547		
ProteinFolding_13	100 ; 1055	21	92	24.5	24.56	-143.3	-143.3	-143.3	-143.3	-143.3	-143.3	-143.3	-143.3	-143.3	
ProteinFolding_14	80 ; 847	20	49	12.6	12.61	-331.7	-331.7	-331.7	-331.7	-331.7	-331.7	-331.7	-331.7	-331.7	
ProteinFolding_15	50 ; 536	22	47	10	9.97	-51.6	-51.6	-51.6	-51.6	-51.6	-51.6	-51.6	-51.6	-51.6	
Segmentation_11	228 ; 852	19	21	6.4	135.02	-152.1	-132.1	-132.3	-132.3	-133.3	-132.3	-132.3	-132.3	-133.4	
Segmentation_12	231 ; 856	20	2	2.9	2.88	-36.7	-21.9	-21.9	-21.9	-30.1	-21.9	-21.9	-21.9	-21.9	
Segmentation_13	225 ; 832	18	2	3.8	3.76	-28.9	-21.4	-21.4	-21.4	-21.4	-21.4	-21.4	-21.4	-21.4	
Segmentation_14	231 ; 863	18	2	3	2.96	-40.2	-39.3	-39.3	-39.3	-40.2	-39.3	-39.3	-39.3	-39.8	
Segmentation_15	229 ; 851	18	21	6.9	336.2	-182.9	-169.6	-163.8	-163.8	-168.5	-163.8	-163.8	-163.8	-168.8	
Segmentation_16	228 ; 838	18	2	2.9	2.94	-42.5	-40.4	-40.4	-40.4	-40.4	-40.4	-40.4	-40.4	-40.8	
Segmentation_17	225 ; 837	20	21	10.2	23.49	-184.1	-176.2	-174.3	-174.3	-175.3	-174.3	-174.3	-174.3	-175.7	
Segmentation_18	235 ; 882	20	2	3	3.04	-44.8	-34	-34	-34	-35.9	-34	-34	-34	-34	
Segmentation_19	228 ; 852	20	2	3.1	3.06	-32.6	-24.1	-24.1	-24.1	-25.4	-24.1	-24.1	-24.1	-24.1	
Segmentation_20	232 ; 867	21	21	8.2	3199.8	-138.1	-112	-112	-112	-112.4	-112	-112	-112	-113.5	
wcsp_14	301 ; 19161	48	8	68.1	75.36	1.5	-29.2	-9.6	1.1	-402.4	1.1	1.1	1.1	-20.9	
wcsp_15	125 ; 736	66	4	16.3	3610.4	-162	-163.6	-80	-74.6	-1503.1	-75.4	-78.6	-83.4	-193.8	
wcsp_16	200 ; 1970	57	44	8.9	3608.9		-5.3	4.8	23.1	-251.5	22.9	22.8	23.2	-8.5	
wcsp_17	340 ; 3417	95	44	12.1	3612.1		-101.8	8.3	33.6	-549.1	34.9	35	34.8		
wcsp_18	239 ; 18016	38	24	80.4	80.47	1.1	-540	-24.1	0.2	-16	0.2	0.2	0.2	-16.9	

Table 2: AOBB-UFO on UAI 2022 Competition Final Problems (3600s)

Summary Statistics

- Total number of problems = 100
- Number of problems for which AOBB-UFO equalized or did better than all of the competing solvers = 34
- Average number of competing solvers AOBB-UFO equalized or did better than = 3.62
- Number of problems for which AOBB-UFO did strictly better than all of the competing solvers = 18
- Average number of competing solvers AOBB-UFO did strictly better than = 2.24
- Number of problems for which AOBB-UFO terminated before 120 seconds = 42

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