ICS 6A
Solution to Homework Assignment 4

Winter 2004

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Answer the following questions (explain your answers).

1. Rosen, page 361, problem 10.
   Answer: $\frac{C(52-2,5-2)}{C(52,5)} = 0.0075$

2. Rosen, page 391, problem 22.
   Answer: The number of positive integers not exceeding 100 is 100, and they are $1, 2, ...100$. The number of positive integers not exceeding 100 and are divisible by 3 is 33, and they are $3 \times 1, 3 \times 2, ...3 \times 33$. So the probability is $\frac{33}{100} = 0.33$.

3. Rosen, page 362, problem 36.
   Answer:

   - the probability of rolling a total of 8 when two dice are rolled: $\frac{5}{6 \times 6} = 0.139$. (Because $8 = 6+2 = 5+3 = 4+4 = 3+5 = 2+6 \to 5$ possibilities)
   - the probability of rolling a total of 8 when three dice are rolled: $\frac{21}{6 \times 6 \times 6} = 0.097$. (Because $8 = 1+1+6 = 1+2+5 = ... \to 21$ possibilities)

   $0.139 > 0.097$, so rolling a total of 8 when two dice are rolled is more likely.

4. Rosen, page 376, problem 2.
   Answer: Assume the probability of each outcome except "3" is p, then the probability of the outcome "3" is $2p$. So $5 \times p + 2p = 1 \implies p = \frac{1}{7}$. The probability of the outcome "3" is $2p = \frac{2}{7}$, the probability of each other outcome except "3" is $\frac{1}{7}$.

5. Rosen, page 376, problem 5 There are 6 pairs that some to 7. All but one (4,3) have probability 1/49. (4,3) has probability 4/49. The total probbaility is therefore 9/49.

6. Rosen, page 377, problem 12.
   Proof: From THEOREM 2 on page 264, we have:
   $p(E \cup F) = p(E) + p(F) - p(E \cap F) \implies p(E \cap F) = p(E) + p(F) - p(E \cup F) = 0.8 + 0.6 - p(E \cup F)$.
   For any probability, it cannot exceed 1, so $p(E \cup F) \leq 1$, such that:
   $p(E \cap F) \geq p(E) + p(F) - 1 = 0.8 + 0.6 - 1 = 0.4$.
   Finally, we have $p(E \cap F) \geq 0.4$
   From the definition $P(E \cap F) \geq P(E)$. Since $P(E) = 0.8$ we get that $P(E \cap F) \geq 0.8$.

7. Rosen, page 377, problem 24.
   The probability is 1/16.

8. Rosen, page 377, problem 26.
   $p(E) = 1/2$. $p(F) = 1/2$ and $p(E \cap F) = 1/4$. Since $P(E \cap F) = p(E) \cdot p(F)$ E and F are independent.

9. Rosen, page 377, problem 28.
   a) exactly three boys: $C(5,3) \times 0.51^3 \times (1 - 0.51)^{(5-3)} = 0.318$
   b) at least one boy: $1 - (1 - 0.51)^5 = 0.972$
   c) at least one girl: $1 - 0.51^5 = 0.965$
   d) all children of the same sex: $0.51^5 + (1 - 0.51)^5 = 0.063$

10. Rosen, page 378, problem 34.

a) the probability of no successes: $(1-p)^n$

b) the probability of at least one successes: $1 - result\ of\ a\ =\ 1-(1-p)^n$

c) the probability of at most one successes: $(1-p)^n + C(n,1) \cdot p^1 \cdot (1-p)^{(n-1)}\ =\ (1-p)^n + n \cdot p \cdot (1-p)^{(n-1)}$

d) the probability of at least two successes: $1 - result\ of\ c\ =\ 1-(1-p)^n - n \cdot p \cdot (1-p)^{(n-1)}$