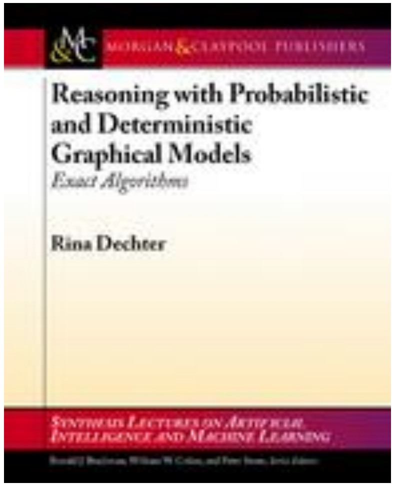
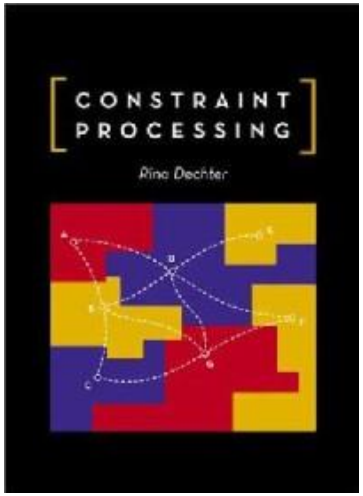
Algorithms for Probabilistic and Deterministic graphical Models

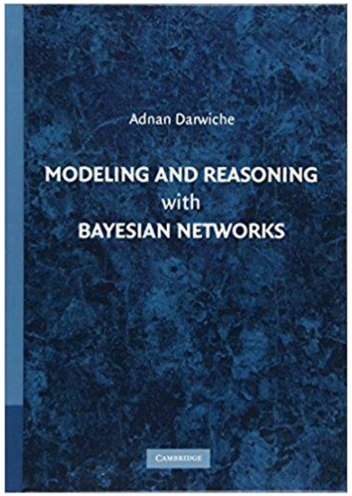
Class 1
Rina Dechter

Dechter-Morgan&claypool book (Dechter 1 book): Chapters 1-2

Text Books


MORGAN & CLAYPOOL PUBLISHERS
Reasoning with Probabilistic and Deterministic Graphical Models
Exact Algorithms
Rina Dechter
SYNTHESIS LECTURES ON ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING
Ronald J. Brachman, William W. Cohen, and Peter Stone, Series Editors


CONSTRAINT PROCESSING
Rina Dechter


Adnan Darwiche
MODELING AND REASONING with BAYESIAN NETWORKS
CAMBRIDGE

Outline

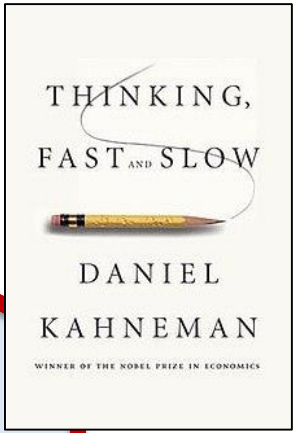
Class page

- Introduction: Constraint and probabilistic graphical models.
- Constraint networks: Graphs, modeling, Inference
- Inference in constraints: Adaptive consistency, constraint propagation, arc-conistency
- Graph properties: induced-width, tree-width, chordal graphs, hypertrees, join-trees
- Bayesian and Markov networks: Representing independencies by graphs
- Building Bayesian networks.
- Inference in Probabilistic models: Bucket-elimination (summation and optimization), Tree-decompositions, Join-tree/Junction-tree algorithm
- Search in CSPs: Backtracking, pruning by constraint propagation, backjumping and learning
- Search in Graphical models: AND/OR search Spaces for likelihood, optimization queries
- Approximate Bounded Inference: weighted Mini-bucket, belief-propagation, generalized belief propagation
- Approximation by Sampling: MCMC schemes, Gibbs sampling, Importance sampling
- Causal Inference with causal graphs.

Course Requirements/Textbook

- Homeworks : There will be 5-6 problem sets , graded 50% of the final grades.
- A term project: paper presentation, a programming project (20%).
- Final (30%)
- Books:
  - “Reasoning with probabilistic and deterministic graphical models”, R. Dechter, Claypool, 2013 https://www.morganclaypool.com/doi/abs/10.2200/S00529ED1V01Y201308AIM023
  - “Modeling and Reasoning with Bayesian Networks”, A. Darwiche, MIT Press, 2009.
  - “Constraint Processing” , R. Dechter, Morgan Kauffman, 2003

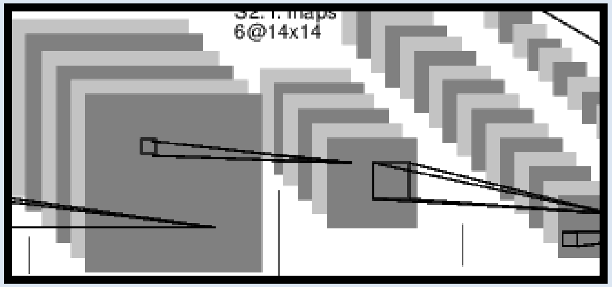
AI Renaissance



THINKING, FAST AND SLOW

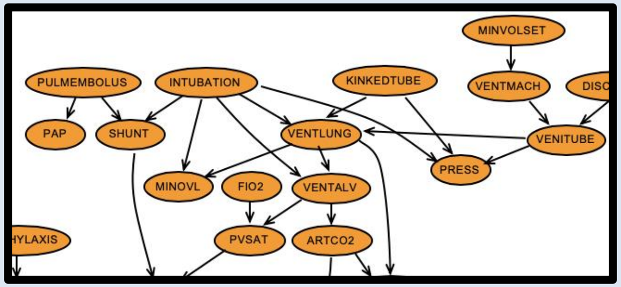
DANIEL KAHNEMAN

WINNER OF THE NOBEL PRIZE IN ECONOMICS



- Deep learning
  - Fast predictions
  - “Instinctive”

Tools:
Tensorflow, PyTorch, …

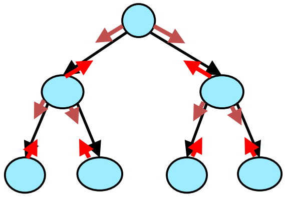


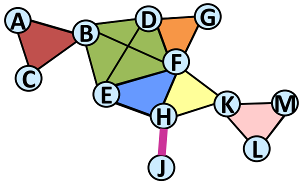
- Probabilistic models
  - Slow reasoning
  - “Logical / deliberative”

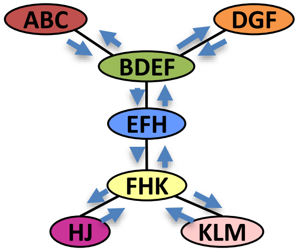
Tools:
Graphical Models,
Probabilistic programming,
Markov Logic, …

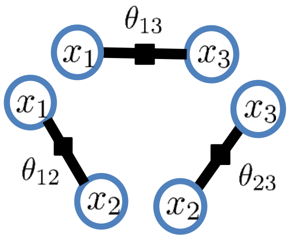
Outline of classes

- Part 1: Introduction and Inference

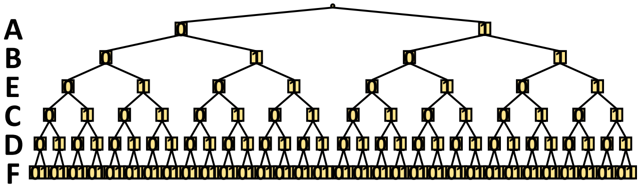


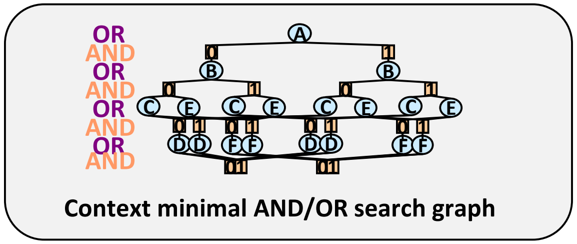




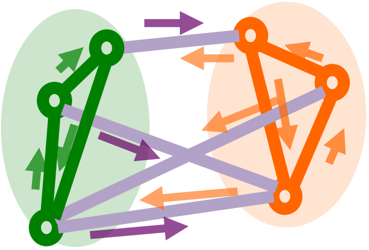


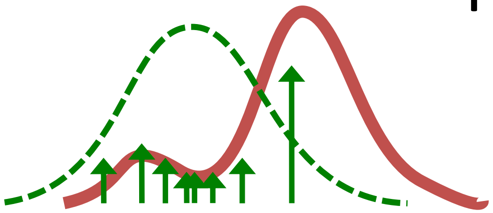
- Part 2: Search





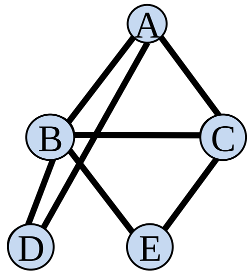
- Parr 3: Variational Methods and Monte-Carlo Sampling





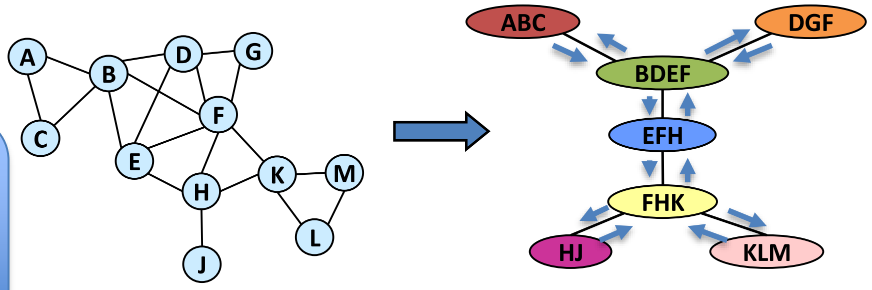
RoadMap: Introduction and Inference

- Basics of graphical models
  - Queries
  - Examples, applications, and tasks
  - Algorithms overview

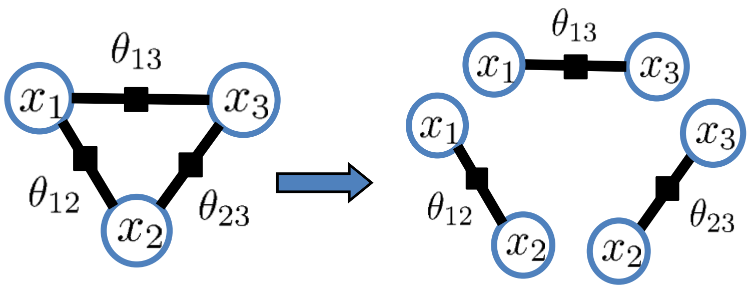


- Inference algorithms, exact
  - Bucket elimination for trees
  - Bucket elimination

For Constraints first



- Approximate elimination
  - Decomposition bounds
  - Mini-bucket & weighted mini-bucket
  - Belief propagation



- Summary and Part 2

RoadMap: Introduction and Inference

- Basics of graphical models
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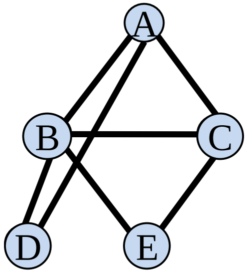
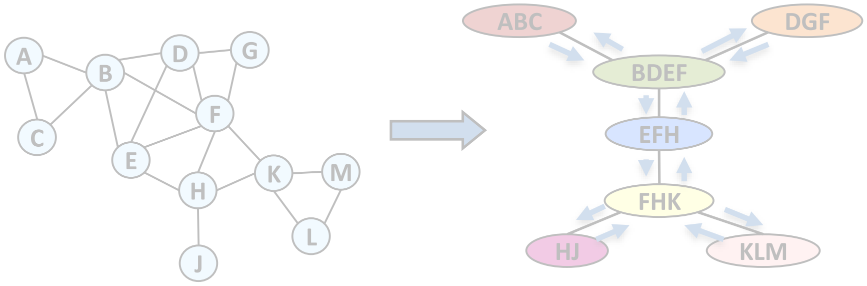


- Inference algorithms, exact
  - Bucket elimination for trees
  - Bucket elimination
  - Jointree clustering
  - Elimination orders



- Approximate elimination
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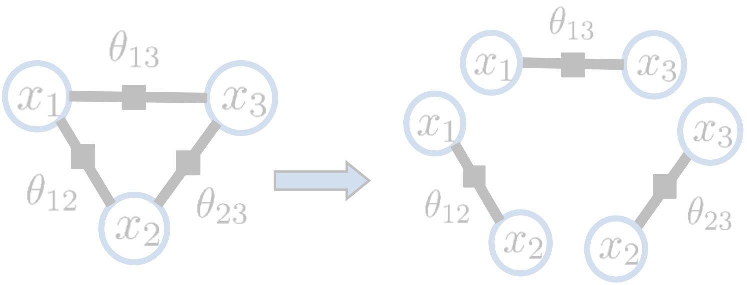


- Summary and Class 2

Probabilistic Graphical models

- Describe structure in large problems
  - Large complex system $F(X)$
  - Made of “smaller”, “local” interactions $f_\alpha(x_\alpha)$
  - Complexity emerges through interdependence

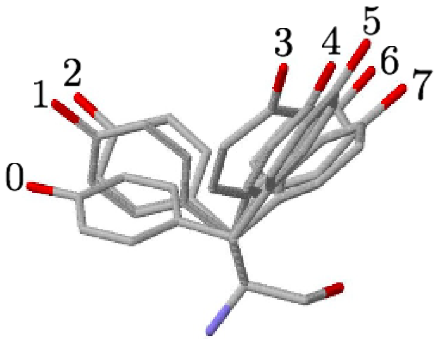
Probabilistic Graphical models

- Describe structure in large problems
  - Large complex system $F(X)$

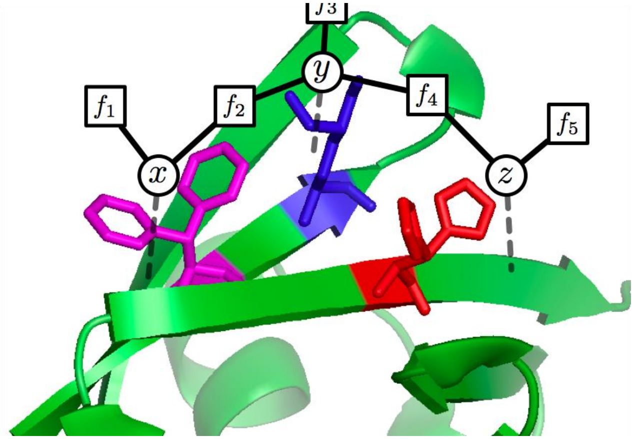
- Protein Structure **prediction**: predicting the 3d structure from given sequences
- PDB: Protein **design** (backbone) algorithms enumerate a combinatorial number of candidate structures to compute the Global Minimum Energy Conformation (GMEC).

-

$$\mathbf{x}^* = \arg\max_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha) \qquad f(\mathbf{x}^*) = \max_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$$



Phenylalanine



[Yanover & Weiss 2002]
[Bruce R. Donald et. Al. 2016]

Probabilistic Graphical models

- Describe structure in large problems
  - Large complex system $F(X)$
  - Made of “smaller”, “local” interactions $f_\alpha(x_\alpha)$
  - Complexity emerges through interdependence
- Examples & Tasks
  - Summation & marginalization

“partition function”

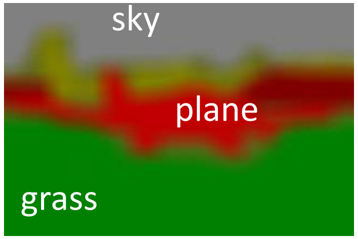
$$p(x_i) = \frac{1}{Z} \sum_{\mathbf{x} \backslash x_i} \prod_\alpha f_\alpha(\mathbf{x}_\alpha) \quad \text{and} \quad Z = \sum_{\mathbf{x}} \prod_\alpha f_\alpha(\mathbf{x}_\alpha)$$

Image segmentation and classification:

Observation **y**

$\Rightarrow$

Marginals p(x_i | **y**)



Observation **y**

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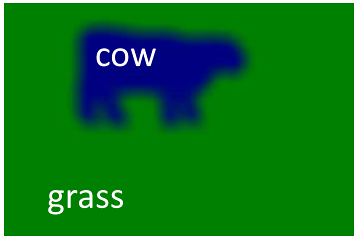
Marginals p(x_i | **y**)



e.g., [Plath et al. 2009]

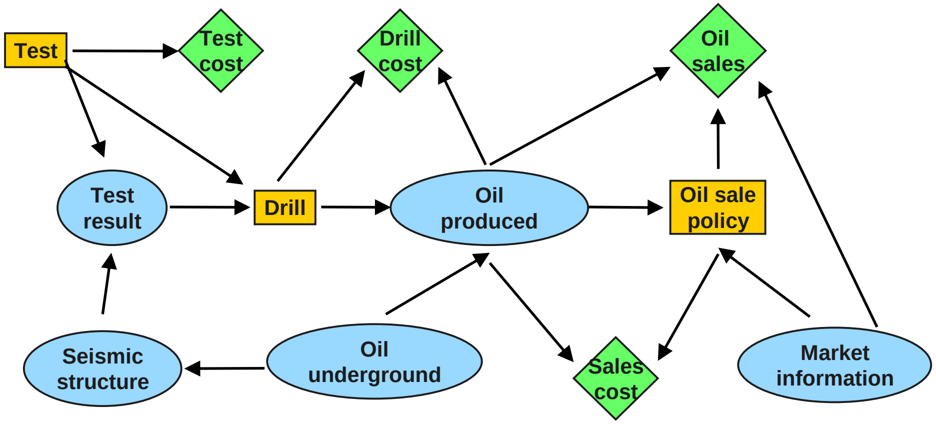
Graphical models

- Describe structure in large problems
  - Large complex system $F(X)$
  - Made of "smaller", "local" interactions $f_\alpha(x_\alpha)$
  - Complexity emerges through interdependence
- Examples & Tasks
  - Mixed inference (marginal MAP, MEU, …)

$$f(\mathbf{x}_M^*) = \max_{\mathbf{x}_M} \sum_{\mathbf{x}_S} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$$

Influence diagrams &
optimal decision-making

(the "oil wildcatter" problem)



e.g., [Raiffa 1968; Shachter 1986]

In more details…

Constraint Networks

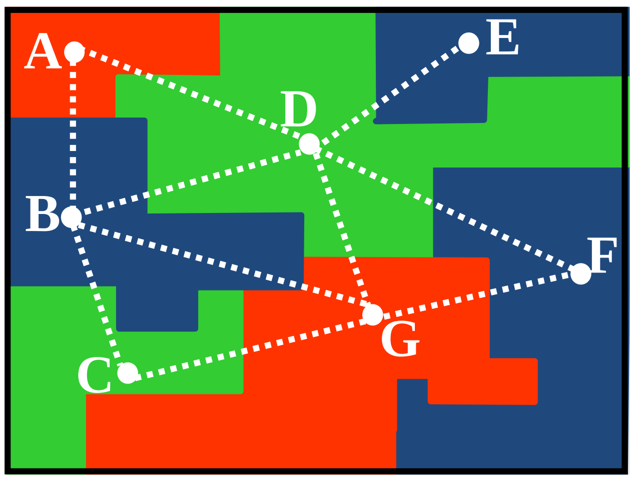
Example: map coloring

Variables - countries (A,B,C,etc.)

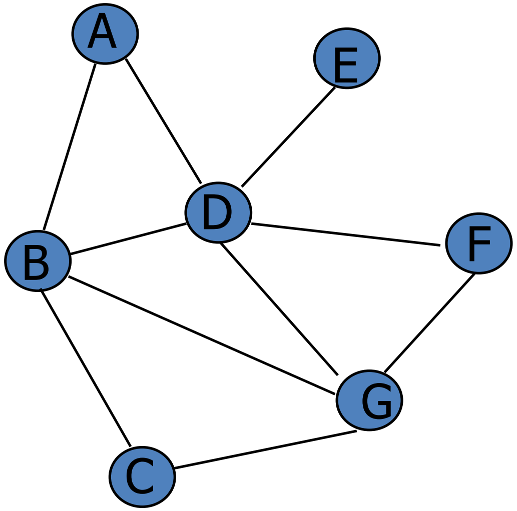
Values - colors (red, green, blue)

Constraints: $A \neq B$, $A \neq D$, $D \neq E$, *etc.*

A	B
red	green
red	yellow
green	red
green	yellow
yellow	green
yellow	red



Constraint graph



Propositional Reasoning

Example: party problem

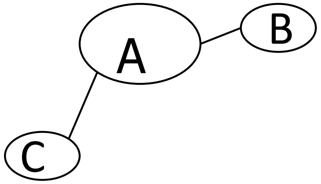
= A = B

- If Alex goes, then Becky goes: $\mathbf{A} \to \mathbf{B}$
- If Chris goes, then Alex goes: $\mathbf{C} \to \mathbf{A}$

= C = A

- **Question:**

Is it possible that Chris goes to the party but Becky does not?

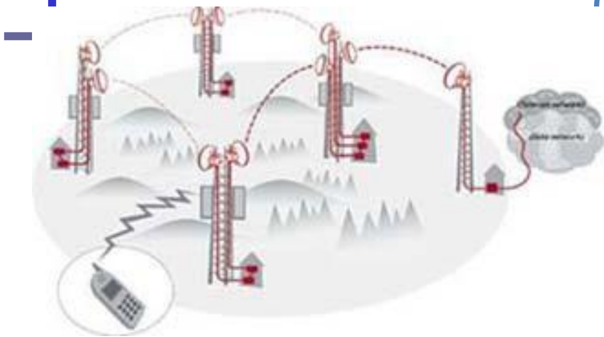


Is the *propositional theory*

$$\varphi = \{A \to B, C \to A, \neg\mathbf{B}, \mathbf{C}\}$$ satisfiable?

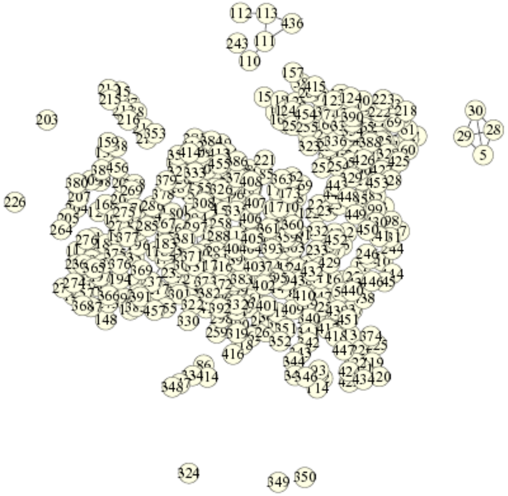
Radio Link Frequency Assignment Problem

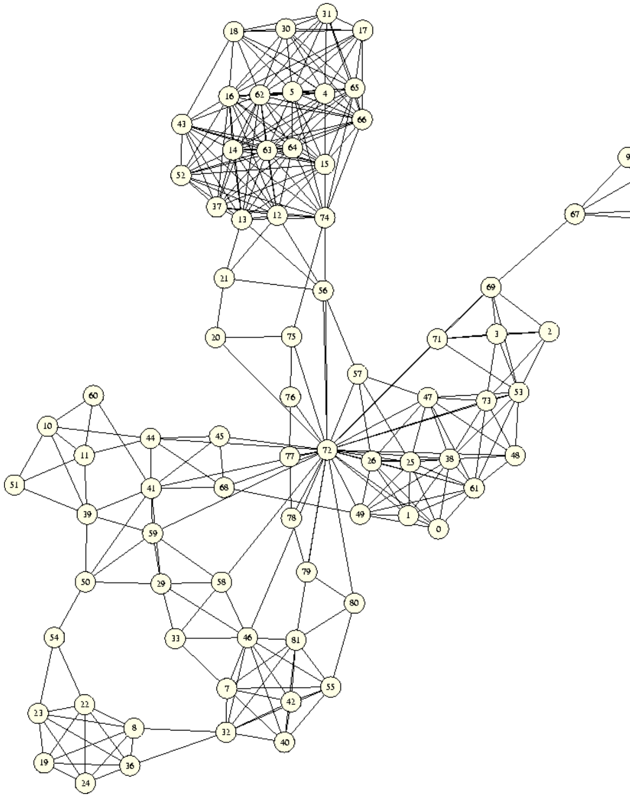
(Cabon et al., *Constraints* 1999) (Koster *et al.*, *4OR* 2003)

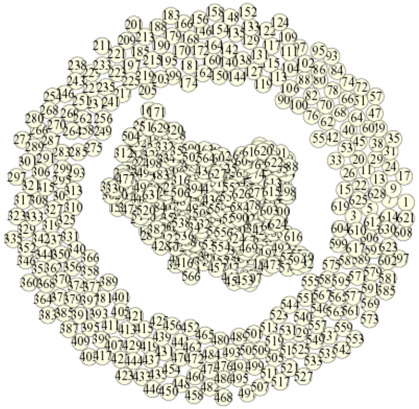


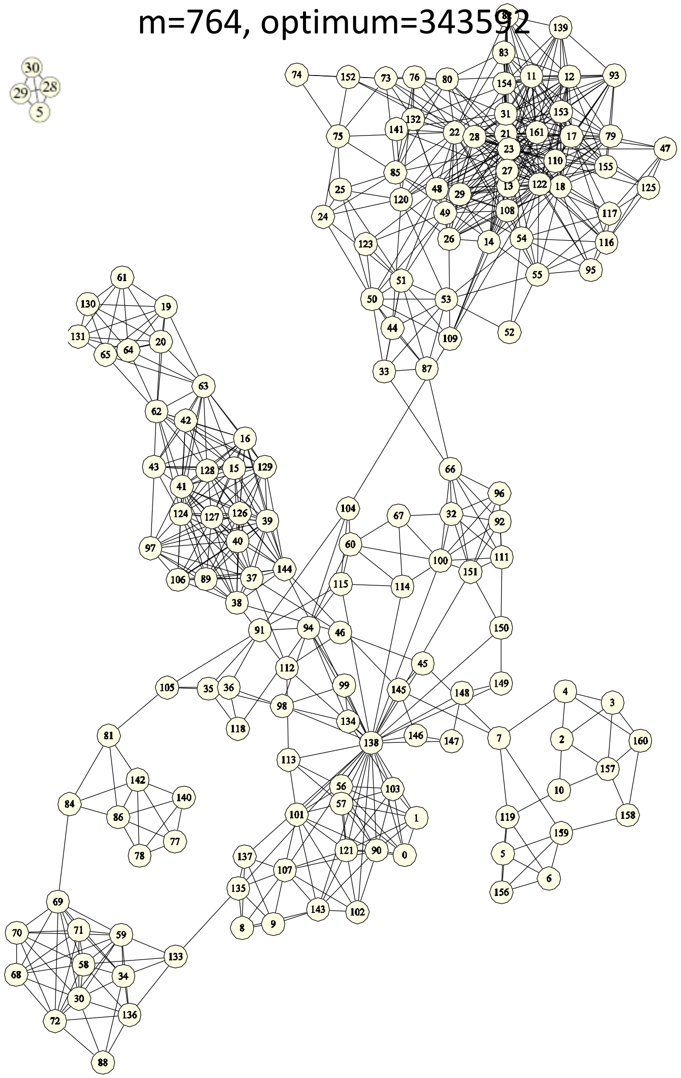
CELAR SCEN-06
n=100, d=44,
m=350, optimum=3389

- CELAR SCEN-07r
  n=162, d=44,
  m=764, optimum=343592



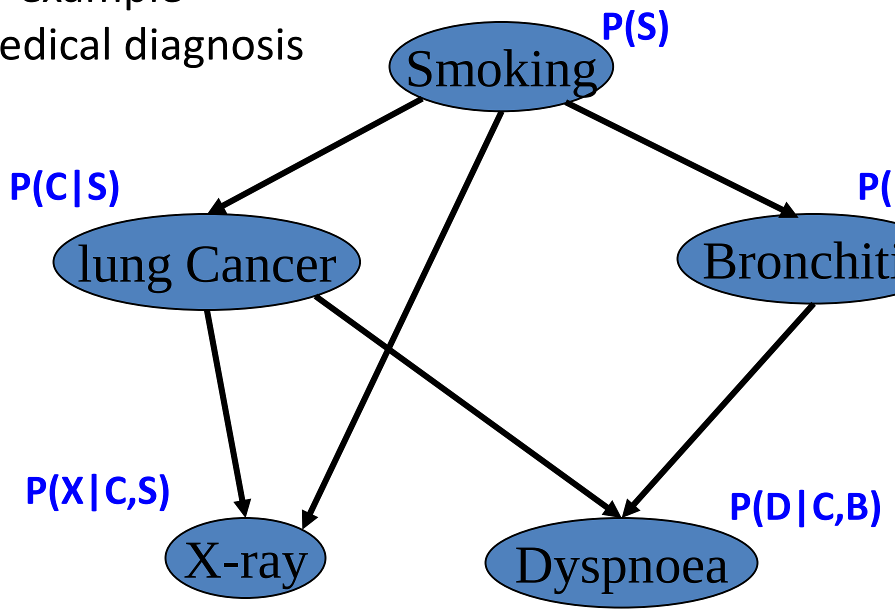






Bayesian Networks (Pearl 1988)

An early example
From medical diagnosis



$$\mathbf{BN} = (\mathbf{G}, \boldsymbol{\Theta})$$

CPD:

C	B	P(D|C,B)
0	0	0.1 0.9
0	1	0.7 0.3
1	0	0.8 0.2
1	1	0.9 0.1

P(S, C, B, X, D) = $P(S)\, P(C|S)\, P(B|S)\, P(X|C,S)\, P(D|C,B)$

Combination: Product
Marginalization: sum/max

- Posterior marginals, probability of evidence, MPE

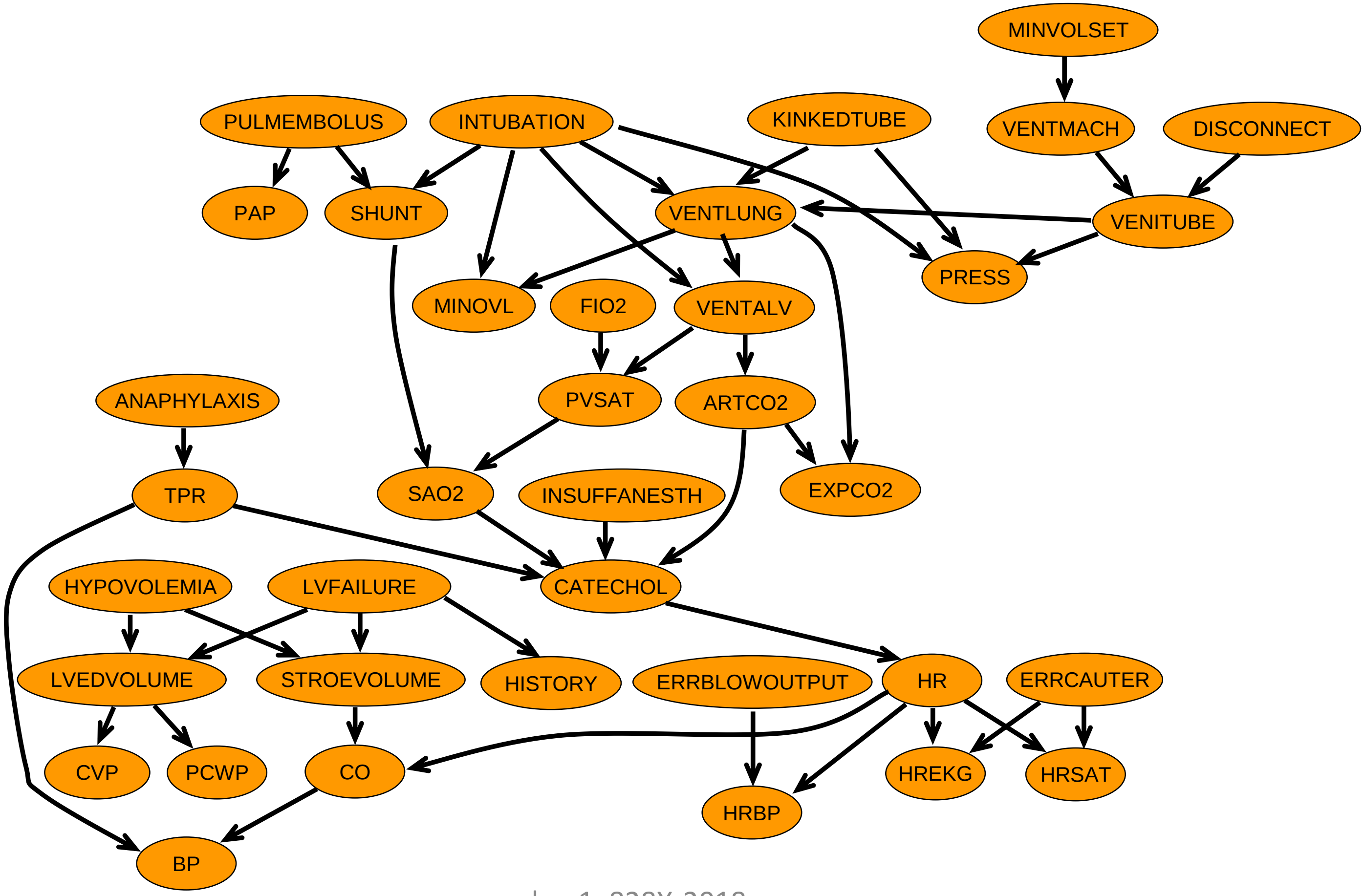
- P(D= 0) = $\sum_{S,L,B,X}$ P(S)· P(C|S)· P(B|S)· P(X|C,S)· P(D|C,B

MAP(P)= $max_{S,L,B,X}$ P(S)· P(C|S)· P(B|S)· P(X|C,S)· P(D|C,B)

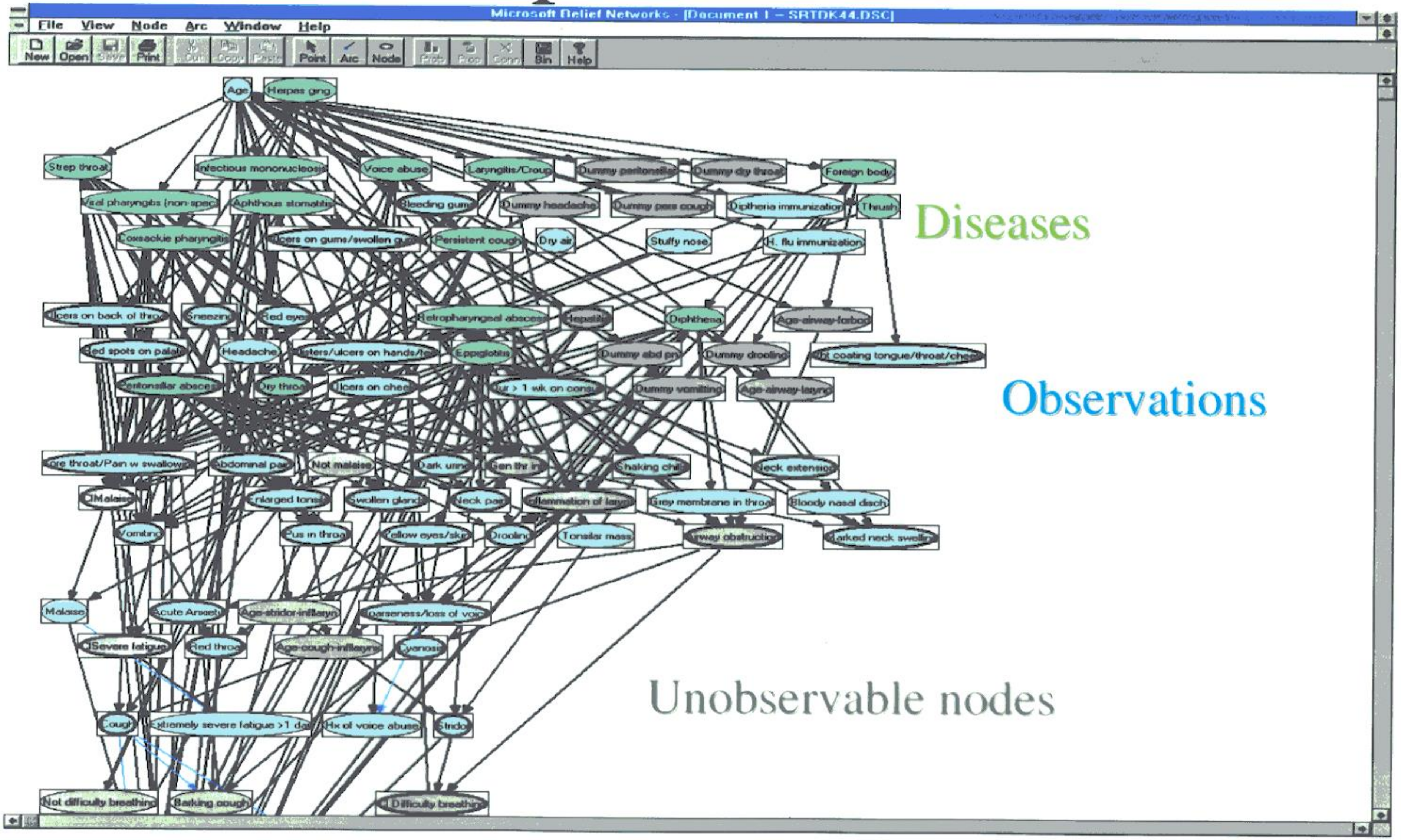
Alarm network [Beinlich et al., 1989]

- Bayes nets: compact representation of large joint distributions

The “alarm” network: 37 variables, 509 parameters (rather than 2^{37} = 10^{11} !)



Chief Complaint: Sore Throat



Diseases

Observations

Unobservable nodes

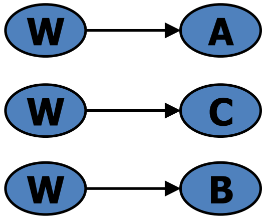
KI

Dechter, Flairs-2018

Probabilistic reasoning (directed)

Party example: the weather effect

- Alex is-likely-to-go in bad weather
- Chris rarely-goes in bad weather
- Becky is indifferent but unpredictable



P(A|W=bad)=.9

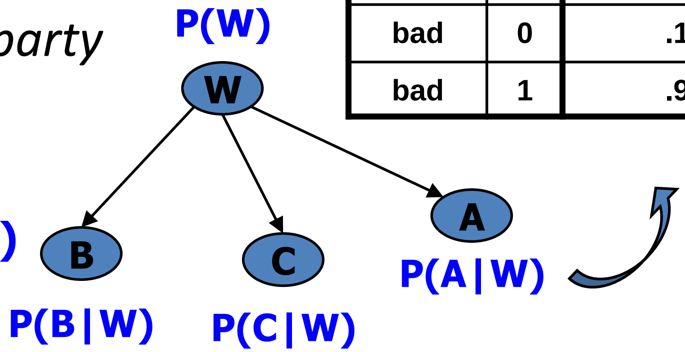
P(C|W=bad)=.1

P(B|W=bad)=.5

Questions:

- *Given bad weather, which group of individuals is most likely to show up at the party?*
- *What is the probability that Chris goes to the party but Becky does not?*

W	A	P(A|W)
good	0	.01
good	1	.99
bad	0	.1
bad	1	.9

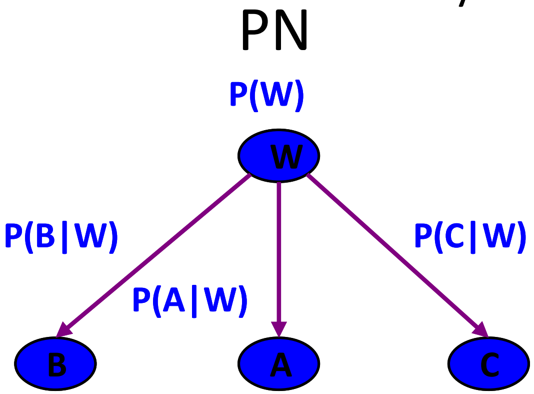


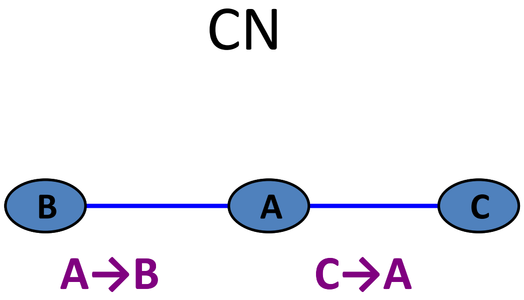
$$P(W,A,C,B) = P(B|W) \cdot P(C|W) \cdot P(A|W) \cdot P(W)$$

$$P(A,C,B|W{=}bad) = 0.9 \cdot 0.1 \cdot 0.5$$

Mixed Probabilistic and Deterministic networks

Alex is-likely-to-go in bad weather
Chris rarely-goes in bad weather
Becky is indifferent but unpredictable





Query:
Is it likely that Chris goes to the party if Becky does not but the weather is bad?

$$P(C, \neg B \mid w = bad, A \rightarrow B, C \rightarrow A)$$

Graphical models (cost networks)

A *graphical model* consists of:

$X = \{X_1, \dots, X_n\}$ -- variables

$D = \{D_1, \dots, D_n\}$ -- domains (we'll assume discrete)

$F = \{f_{\alpha_1}, \dots, f_{\alpha_m}\}$-- functions or "factors"

and a *combination operator*

Example:

$A \in \{0,1\}$

$B \in \{0,1\}$

$C \in \{0,1\}$

$f_{AB}(A,B), \quad f_{BC}(B,C)$

The *combination operator* defines an overall function from the individual factors,

e.g., "+" : $F(A,B,C) = f_{AB}(A,B) + f_{BC}(B,C)$

Notation:

Discrete Xi values called states

Tuple or configuration: states taken by a set of variables

Scope of f: set of variables that are arguments to a factor f

often index factors by their scope, e.g., $f_\alpha(X_\alpha), \quad X_\alpha \subseteq X$

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$f_{AB}(A,B), \quad f_{BC}(B,C)$

$$F(A,B,C) = f_{AB}(A,B) + f_{BC}(B,C)$$

For discrete variables, think of functions as "tables"

(though we might represent them more efficiently)

A	B	f(A,B)
0	0	6
0	1	0
1	0	0
1	1	6

$F(A=0, B=1, C=1)$

+

B	C	f(B,C)
0	0	6
0	1	0
1	0	0
1	1	6

=

A	B	C	f(A,B,C)
0	0	0	12
0	0	1	6
0	1	0	0
0	1	1	6
1	0	0	6
1	0	1	0
1	1	0	6
1	1	1	12

= 0 + 6

Graph Visualiization: Primal Graph

A *graphical model* consists of:

$X = \{X_1, \ldots, X_n\}$ -- variables

$D = \{D_1, \ldots, D_n\}$ -- domains

$F = \{f_{\alpha_1}, \ldots, f_{\alpha_m}\}$ -- functions or "factors"

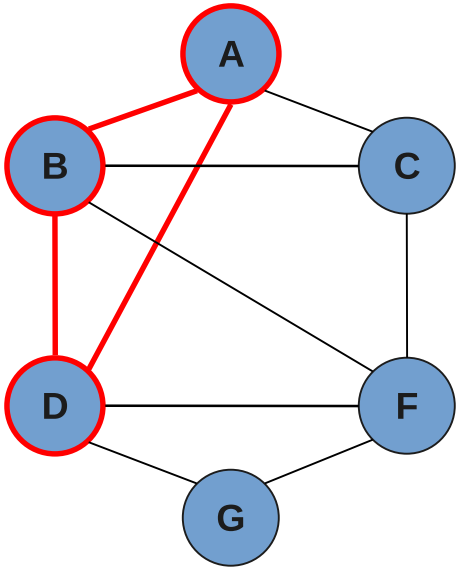
and a *combination operator*

Primal graph:

variables → nodes

factors → cliques

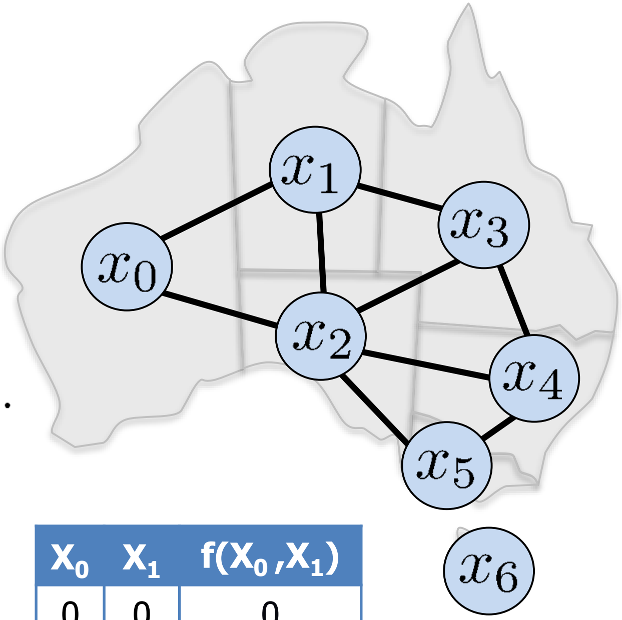
$$F(A,B,C,D,F,G) = f_1(A,B,D) + f_2(D,F,G) + f_3(B,C,F) + f_4(A,C)$$



Example: Constraint networks

$X_i \in \{red, green, blue\}$

$f_{ij}(X_i, X_j) = (X_i \neq X_j)$ for adjacent regions i,j



Overall function is “and” of individual constraints:

$$F(X) = f_{01}(X_0, X_1) \wedge f_{12}(X_1, X_2) \wedge f_{02}(X_0, X_2) \wedge \ldots$$

“Tabular” form:

$$f_{ij}(X_i, X_j) = \begin{cases} 1.0 & X_i \neq X_j \\ 0.0 & X_i = X_j \end{cases}$$

$$F(X) = \prod_{ij} f_{ij}(X_i, X_j) = \begin{cases} 1.0 & \text{all valid} \\ 0.0 & \text{any invalid} \end{cases}$$

X_0	X_1	$f(X_0, X_1)$
0	0	0
0	1	1
0	2	1
1	0	1
1	1	0
1	2	1
2	0	1
2	1	1
2	2	0

Tasks: “max”: is there a solution?
“sum”: how many solutions?

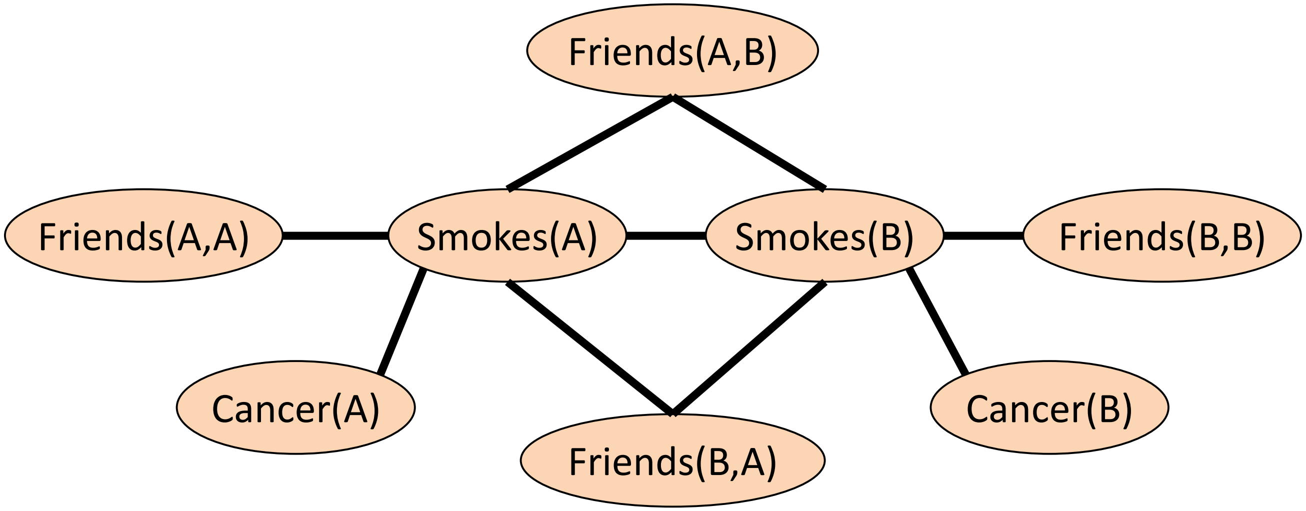
Markov logic, Markov networks

[Richardson & Domingos 2005]

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

S_A	C_A	$f(S_A,C_A)$
0	0	exp(1.5)
0	1	exp(1.5)
1	0	1.0
1	1	exp(1.5)

Two constants: **Anna** (A) and **Bob** (B)



F_{AB}	S_A	S_B	f(.)
0	0	0	exp(1.1)
0	0	1	exp(1.1)
0	1	0	exp(1.1)
0	1	1	exp(1.1)
1	0	0	exp(1.1)
1	0	1	1.0
1	1	0	1.0
1	1	1	exp(1.1)

Graphical visualization

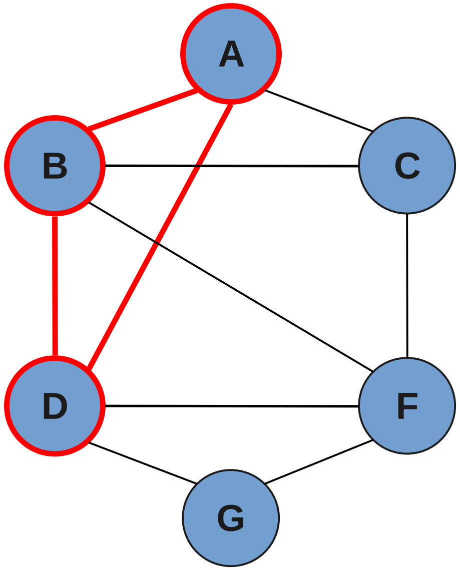
A *graphical model* consists of:

$X = \{X_1, \ldots, X_n\}$ -- variables

$D = \{D_1, \ldots, D_n\}$ -- domains

$F = \{f_{\alpha_1}, \ldots, f_{\alpha_m}\}$-- functions or “factors”

and a *combination operator*

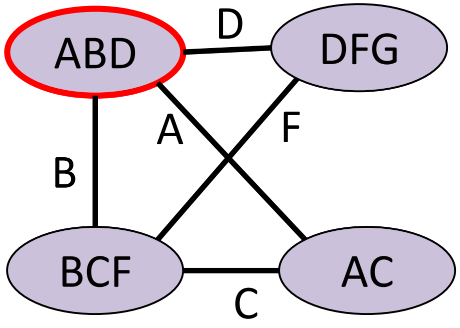


Primal graph:

variables → nodes

factors → cliques

$$F(A,B,C,D,F,G) = f_1(A,B,D) + f_2(D,F,G) + f_3(B,C,F) + f_4(A,C)$$



Dual graph:

factor scopes → nodes

edges → intersections (separators)

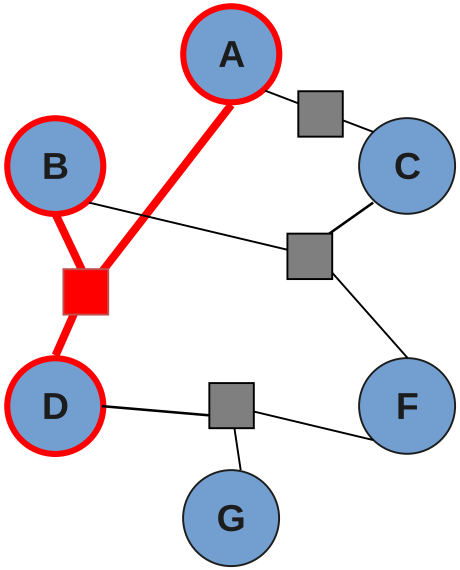
Graphical visualization

“Factor” graph: explicitly indicate the scope of each factor

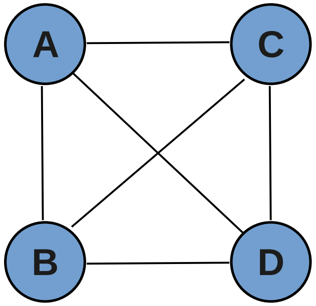
variables → circles

factors → squares

$$F(A,B,C,D,F,G) = f_1(A,B,D) + f_2(D,F,G) \\ + f_3(B,C,F) + f_4(A,C)$$

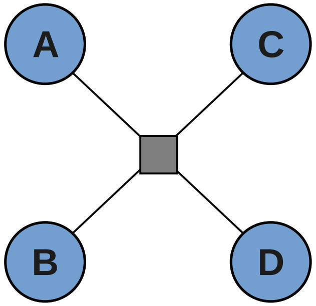


Useful for disambiguating factorization:



$\stackrel{?}{=}$

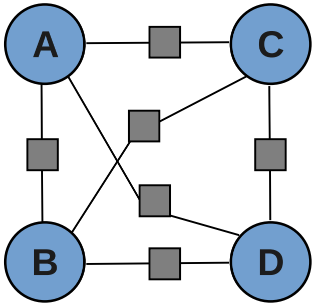
$f_1(A,B,C,D)$



O(d^4)

vs.

$f_1(A,B) + f_2(A,C) + \ldots$



pairwise: O(d^2)

Graphical models

A ***graphical model*** consists of:

$X = \{X_1, \ldots, X_n\}$ -- variables

$D = \{D_1, \ldots, D_n\}$ -- domains

$F = \{f_{\alpha_1}, \ldots, f_{\alpha_m}\}$-- functions or “factors”

Operators:

combination operator

(sum, product, join, …)

elimination operator

(projection, sum, max, min, ...)

Types of queries:

Marginal: $Z = \sum_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$

MPE / MAP: $f(\mathbf{x}^*) = \max_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$

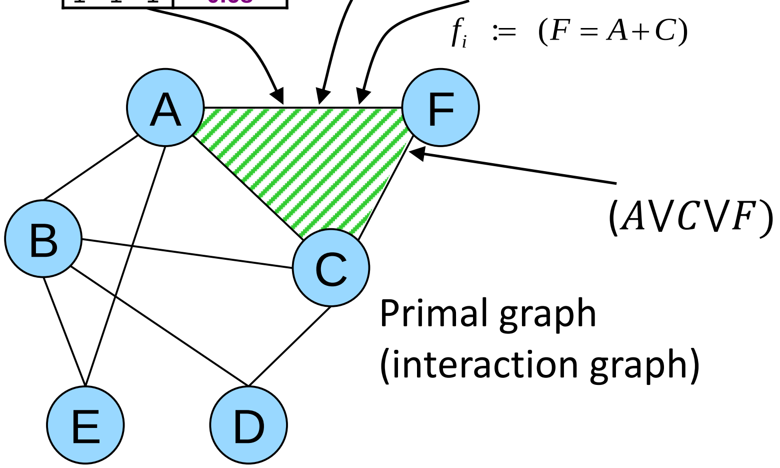
Marginal MAP: $f(\mathbf{x}_M^*) = \max_{\mathbf{x}_M} \sum_{\mathbf{x}_S} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$

Conditional Probability Table (CPT)

A	C	F	P(F|A,C)
0	0	0	0.14
0	0	1	0.96
0	1	0	0.40
0	1	1	0.60
1	0	0	0.35
1	0	1	0.65
1	1	0	0.72
1	1	1	0.68

Relation

A	C	F
red	green	blue
blue	red	red
blue	blue	green
green	red	blue



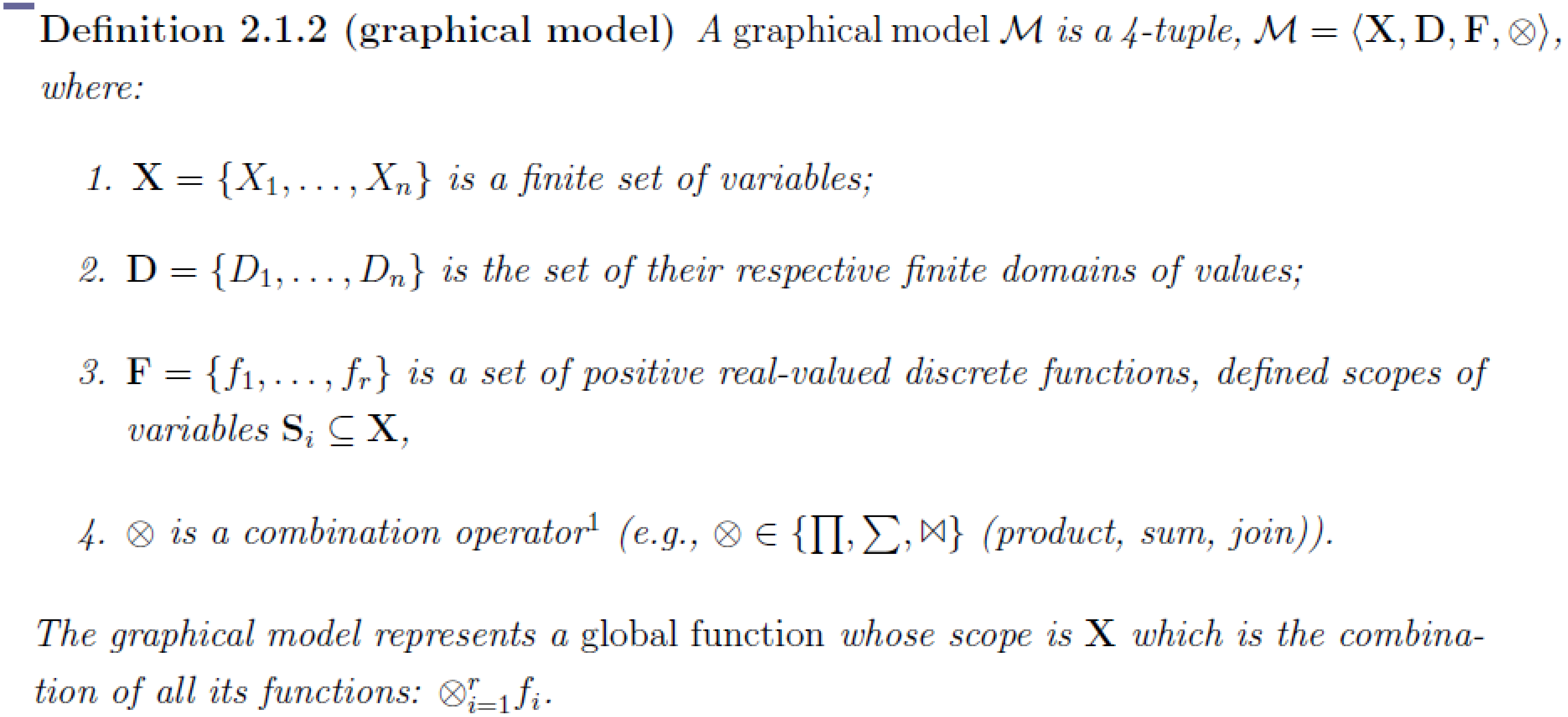
$f_i := (F = A + C)$

$(A \vee C \vee F)$

Primal graph (interaction graph)

- All these tasks are NP-hard
  - exploit problem structure
  - identify special cases
  - approximate

Graphical models/reasoning task



Definition 2.1.2 (graphical model) *A* graphical model $\mathcal{M}$ *is a 4-tuple,* $\mathcal{M} = \langle \mathbf{X}, \mathbf{D}, \mathbf{F}, \otimes \rangle$, *where:*

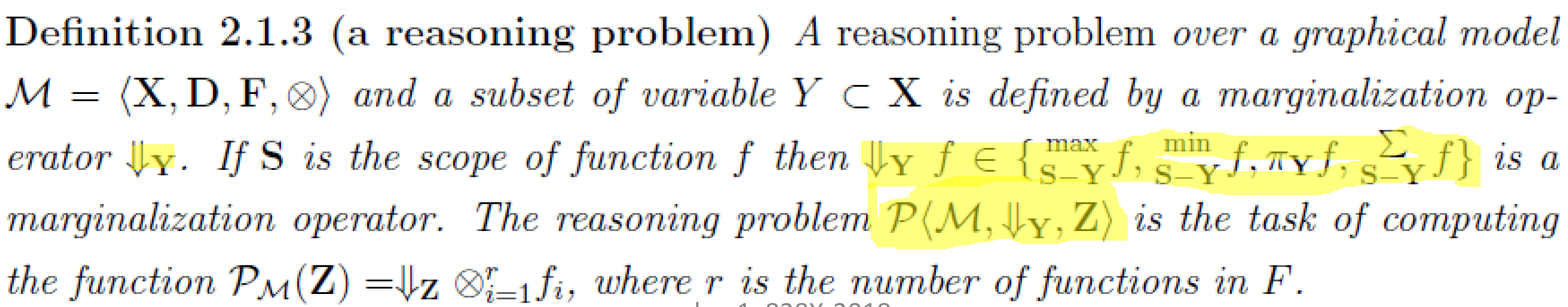
1. $\mathbf{X} = \{X_1, \ldots, X_n\}$ *is a finite set of variables;*

2. $\mathbf{D} = \{D_1, \ldots, D_n\}$ *is the set of their respective finite domains of values;*

3. $\mathbf{F} = \{f_1, \ldots, f_r\}$ *is a set of positive real-valued discrete functions, defined scopes of variables* $\mathbf{S}_i \subseteq \mathbf{X}$,

4. $\otimes$ *is a combination operator*[1] *(e.g.,* $\otimes \in \{\prod, \sum, \bowtie\}$ *(product, sum, join)).*

The graphical model represents a global function *whose scope is* $\mathbf{X}$ *which is the combination of all its functions:* $\otimes_{i=1}^{r} f_i$.



Definition 2.1.3 (a reasoning problem) *A* reasoning problem *over a graphical model* $\mathcal{M} = \langle \mathbf{X}, \mathbf{D}, \mathbf{F}, \otimes \rangle$ *and a subset of variable* $Y \subset \mathbf{X}$ *is defined by a marginalization operator* $\Downarrow_{\mathbf{Y}}$. *If* $\mathbf{S}$ *is the scope of function* f *then* $\Downarrow_{\mathbf{Y}} f \in \{\max_{\mathbf{S}-\mathbf{Y}} f, \min_{\mathbf{S}-\mathbf{Y}} f, \pi_{\mathbf{Y}} f, \sum_{\mathbf{S}-\mathbf{Y}} f\}$ *is a marginalization operator. The reasoning problem* $\mathcal{P}\langle \mathcal{M}, \Downarrow_{\mathbf{Y}}, \mathbf{Z}\rangle$ *is the task of computing the function* $\mathcal{P}_{\mathcal{M}}(\mathbf{Z}) = \Downarrow_{\mathbf{Z}} \otimes_{i=1}^{r} f_i$, *where* r *is the number of functions in* F.

Summary of graphical models types

- Constraint networks
- Cost networks
- Bayesian network
- Markov networks
- Mixed probability and constraint network
- Influence diagrams

Constraint Networks

Map coloring

Combination = join

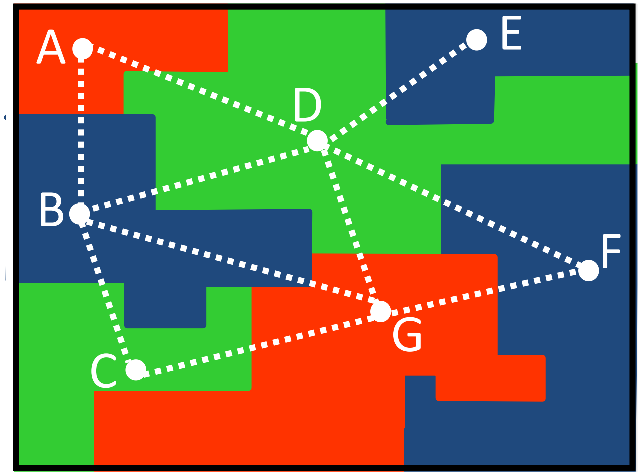
Marginalization = projection

Variables: countries (A B C etc.)

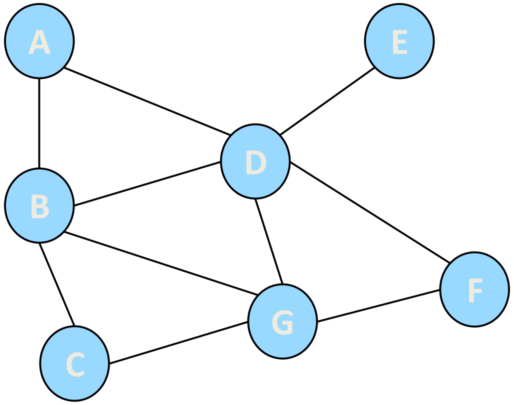
Values: colors (red green blue)

Constraints: $\mathbf{A \neq B, A \neq D, D \neq E, \ldots}$

A	B
red	green
red	yellow
green	red
green	yellow
yellow	green
yellow	red

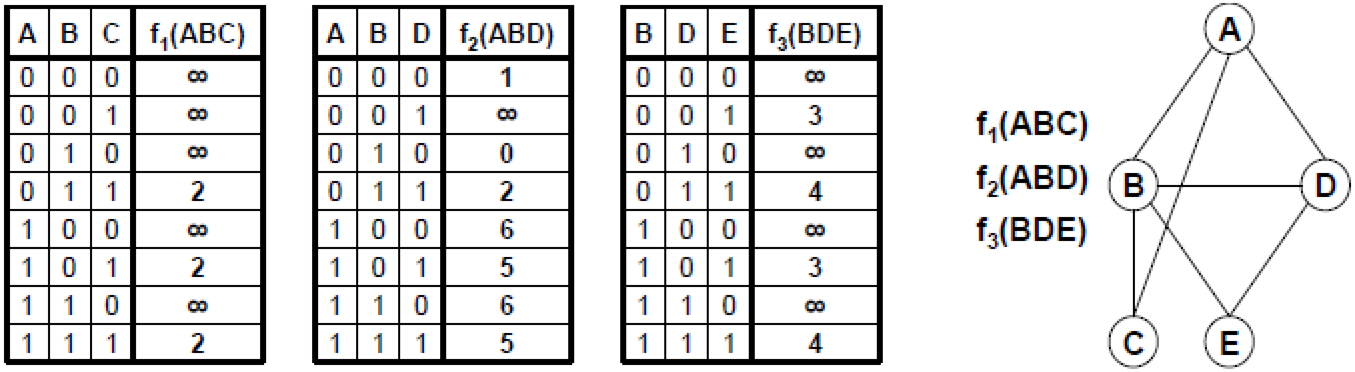


Constraint graph



Queries: Find one solution, all solutions, counting

Example of a Cost Network



A	B	C	$f_1(ABC)$
0	0	0	∞
0	0	1	∞
0	1	0	∞
0	1	1	2
1	0	0	∞
1	0	1	2
1	1	0	∞
1	1	1	2

A	B	D	$f_2(ABD)$
0	0	0	1
0	0	1	∞
0	1	0	0
0	1	1	2
1	0	0	6
1	0	1	5
1	1	0	6
1	1	1	5

B	D	E	$f_3(BDE)$
0	0	0	∞
0	0	1	3
0	1	0	∞
0	1	1	4
1	0	0	∞
1	0	1	3
1	1	0	∞
1	1	1	4

(a) Cost functions

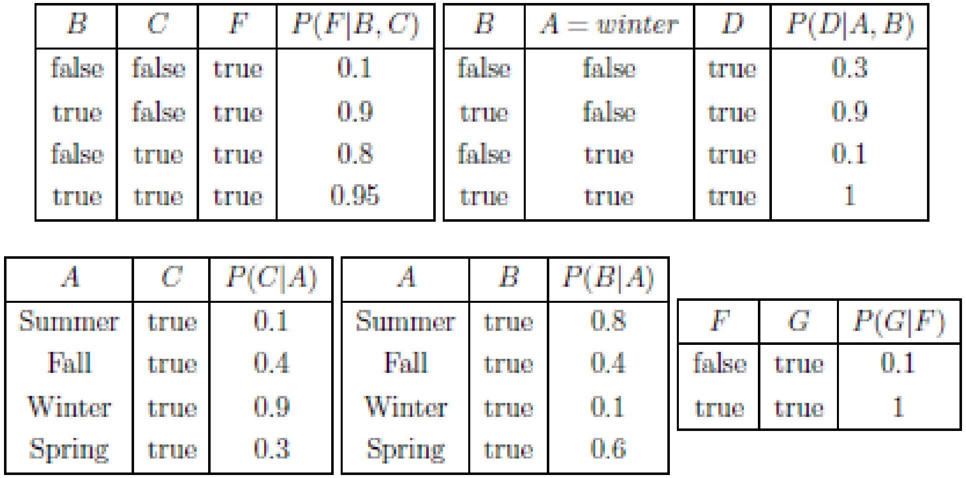
(b) Constraint graph

Figure 2.3: A cost network.

Combination: sum
Marginalization:min/max

Definition 2.3.2 (WCSP) *A Weighted Constraint Satisfaction Problem (WCSP) is a graphical model* $\langle \mathbf{X}, \mathbf{D}, \mathbf{F}, \sum \rangle$ *where each of the functions* $f_i \in \mathbf{F}$ *assigns "0" (no penalty) to allowed tuples and a positive integer penalty cost to the forbidden tuples. Namely,* $f_i : D_{X_{i_1}} \times ... \times D_{X_{i_t}} \rightarrow \mathbb{N}$, *where* $S_i = \{X_{i_1}, ..., X_{i_t}\}$ *is the scope of the function.*

A Bayesian Network



B	C	F	$P(F\|B,C)$
false	false	true	0.1
true	false	true	0.9
false	true	true	0.8
true	true	true	0.95

B	$A = winter$	D	$P(D\|A,B)$
false	false	true	0.3
true	false	true	0.9
false	true	true	0.1
true	true	true	1

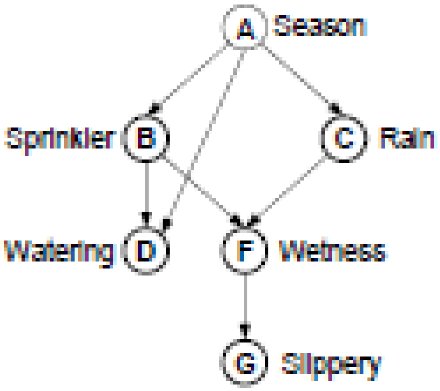
A	C	$P(C\|A)$
Summer	true	0.1
Fall	true	0.4
Winter	true	0.9
Spring	true	0.3

A	B	$P(B\|A)$
Summer	true	0.8
Fall	true	0.4
Winter	true	0.1
Spring	true	0.6

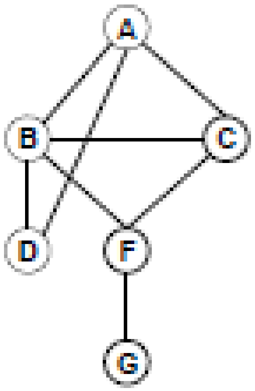
F	G	$P(G\|F)$
false	true	0.1
true	true	1

Combination: product

Marginalization: sum or min/max



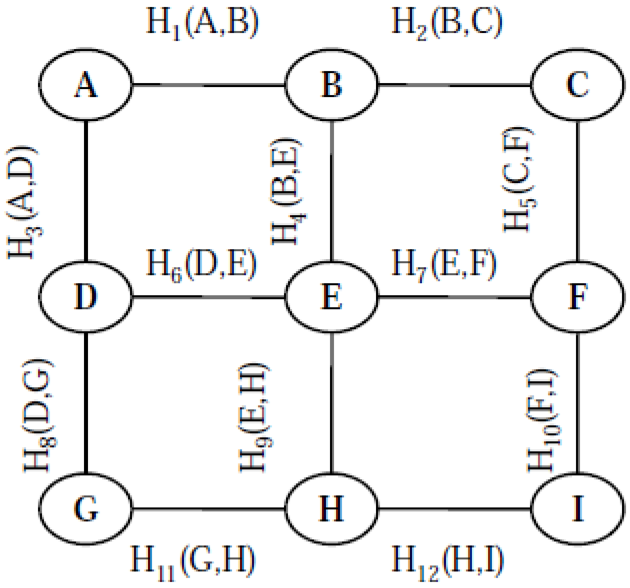
(a) Directed acyclic graph



(b) Moral graph

Belief network $P(g,f,c,b,a) = P(g|f)P(f|c,b)P(d|a,b)P(c|1)P(b|a)P(a)$

Markov Networks



D	E	H_6(D,E)
0	0	20.2
0	1	12
1	0	23.4
1	1	11.7

(a) (b)

Figure 2.6: (a) An example 3 × 3 square Grid Markov network (ising model) and (b) An example potential $H_6(D, E)$

network represents a global joint distribution over the variables $\boldsymbol{X}$ *given by:*

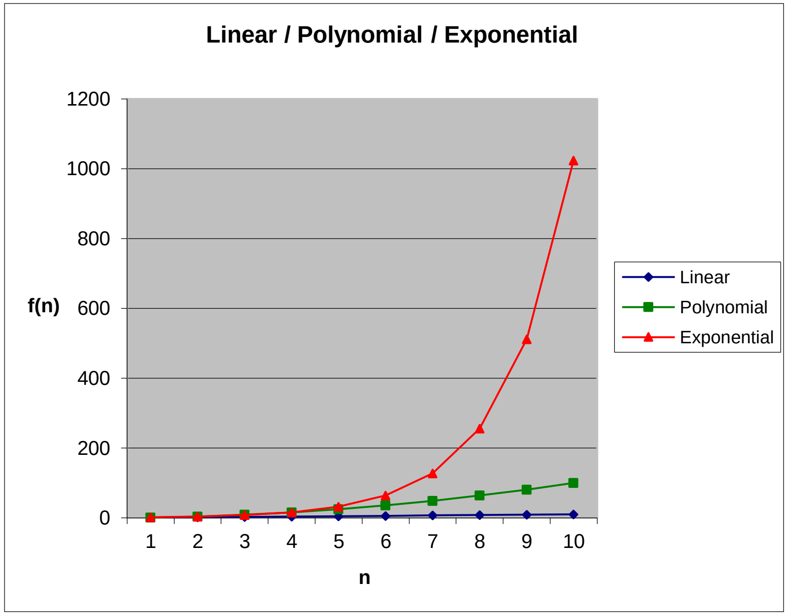
$$P(\boldsymbol{x}) = \frac{1}{Z}\prod_{i=1}^{m} H_i(\boldsymbol{x}) \quad , \quad Z = \sum_{\boldsymbol{x}\in \boldsymbol{X}}\prod_{i=1}^{m} H_i(\boldsymbol{x})$$

Example domains for graphical models

- Natural Language processing
  - Information extraction, semantic parsing, translation, topic models, …
- Computer vision
  - Object recognition, scene analysis, segmentation, tracking, …
- Computational biology
  - Pedigree analysis, protein folding and binding, sequence matching, …
- Networks
  - Webpage link analysis, social networks, communications, citations, ….
- Robotics
  - Planning & decision making

Complexity of Reasoning Tasks

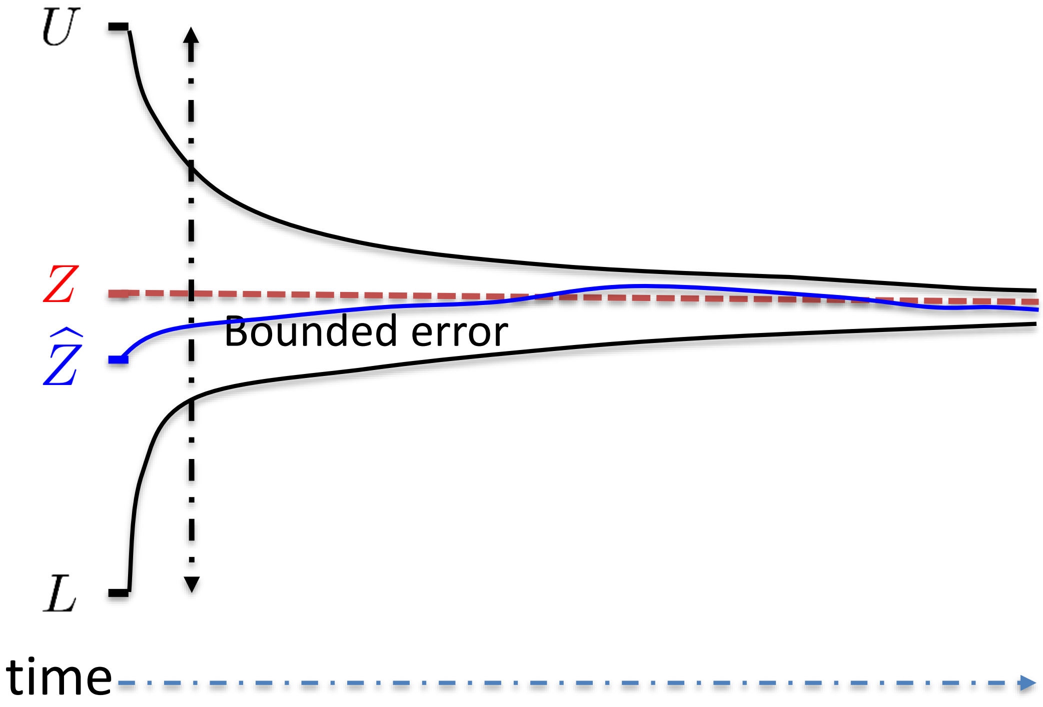
- Constraint satisfaction
- Counting solutions
- Combinatorial optimization
- Belief updating
- Most probable explanation
- Decision-theoretic planning



Reasoning is computationally hard

Complexity is Time and space(memory)

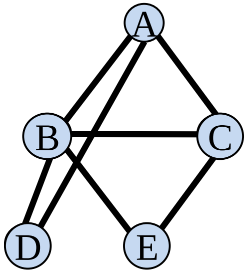
Desired Properties: Guarantee, Anytime, Anyspace



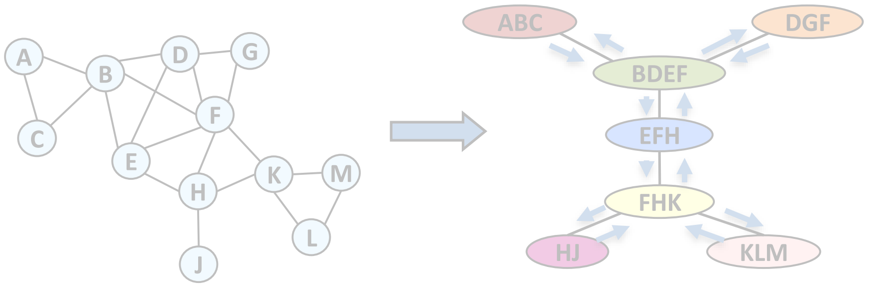
- Anytime
  - valid solution at any point
  - solution quality improves with additional computation
- Anyspace
  - run with limited memory resources

RoadMap: Introduction and Inference

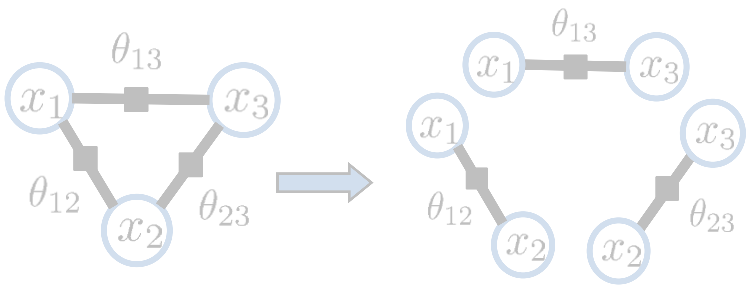
- Basics of graphical models
  - Queries
  - Examples, applications, and tasks
  - **Algorithms overview**



- Inference algorithms, exact
  - Bucket elimination for trees
  - Bucket elimination
  - Jointree clustering
  - Elimination orders



- Approximate elimination
  - Decomposition bounds
  - Mini-bucket & weighted mini-bucket
  - Belief propagation

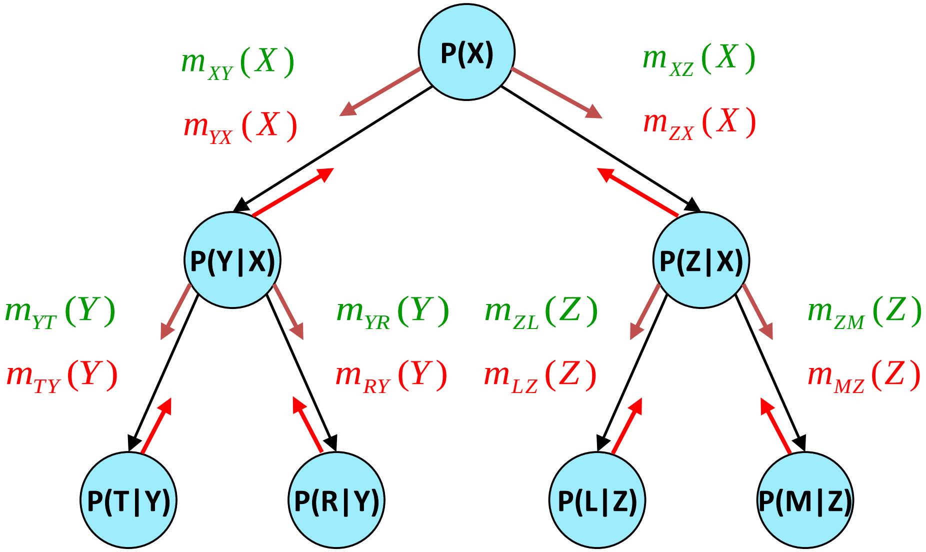


- Summary and Class 2

Tree-solving is easy

Belief updating (sum-prod)

CSP – consistency (projection-join)



MPE (max-prod)

#CSP (sum-prod)

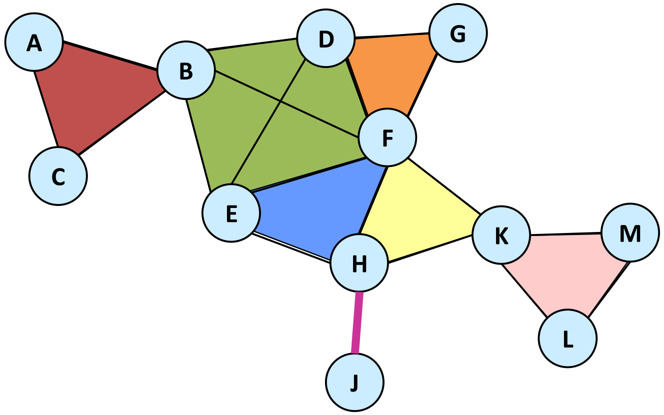
Trees are processed in linear time and memory

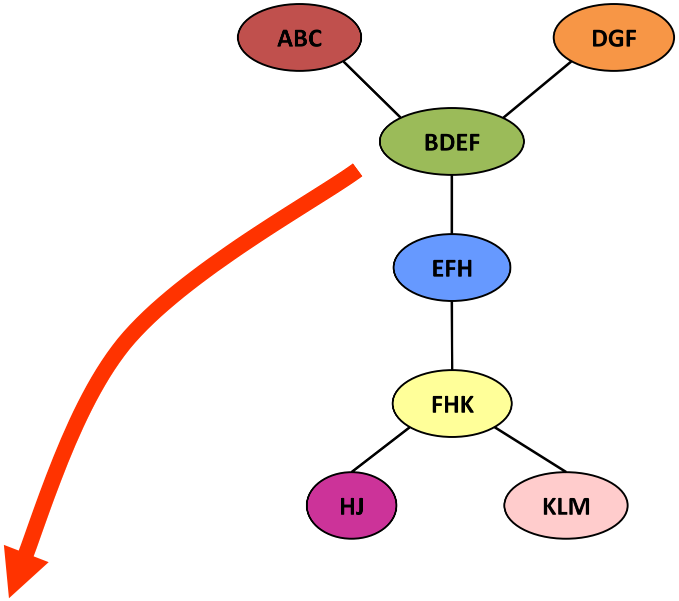
Transforming into a Tree

- **By Inference (thinking)**
  - Transform into a single, equivalent tree of sub-problems

- **By Conditioning (guessing)**
  - Transform into many tree-like sub-problems.

Inference and Treewidth





Inference algorithm:

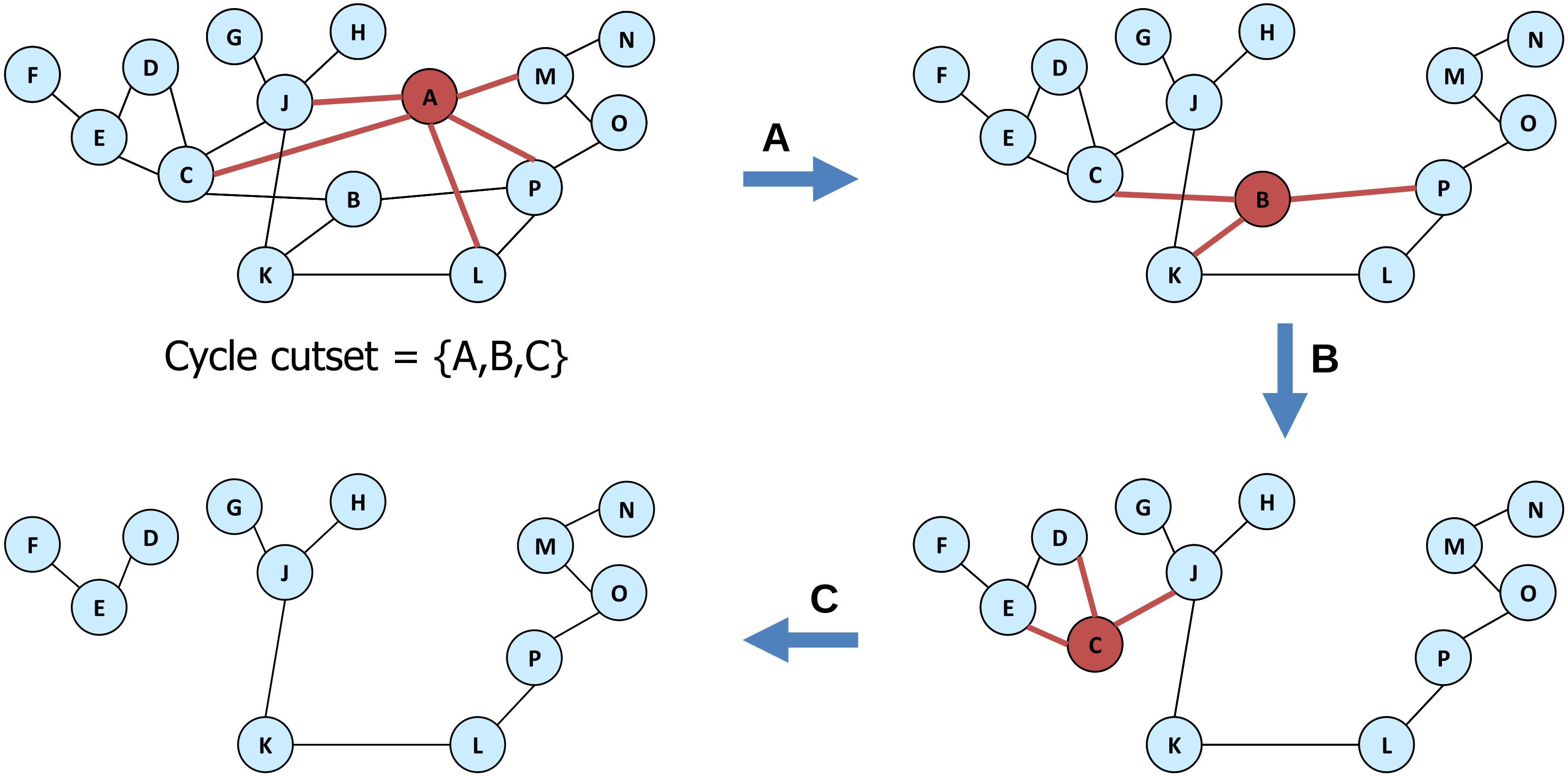
Time: exp(tree-width)

Space: exp(tree-width)

treewidth = 4 - 1 = 3

treewidth = (maximum cluster size) - 1

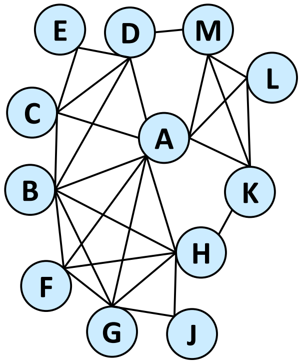
Conditioning and Cycle cutset



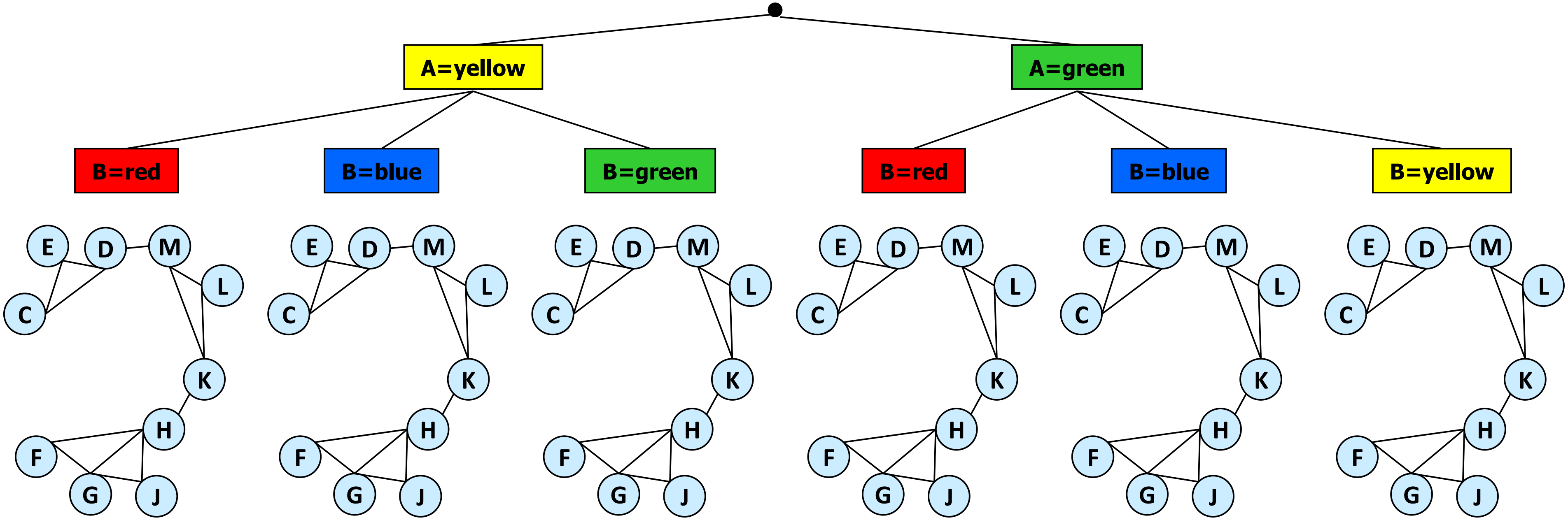
Cycle cutset = {A,B,C}

Search over the Cutset

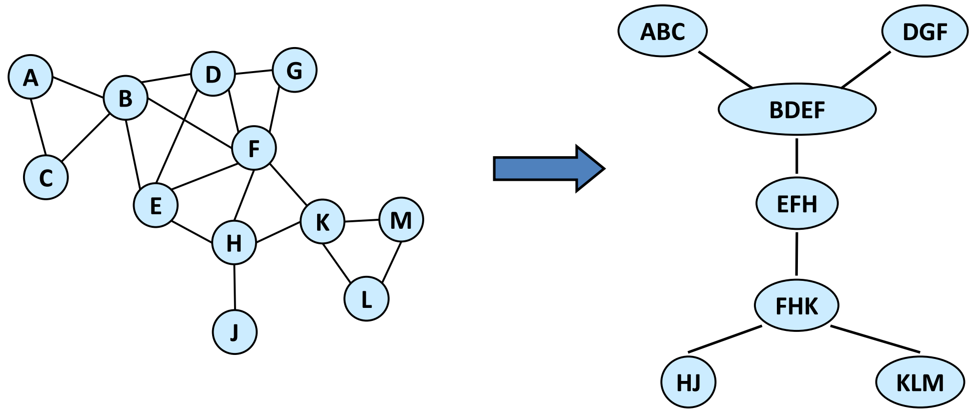
Graph Coloring problem



- Inference may require too much memory
- **Condition** on some of the variables

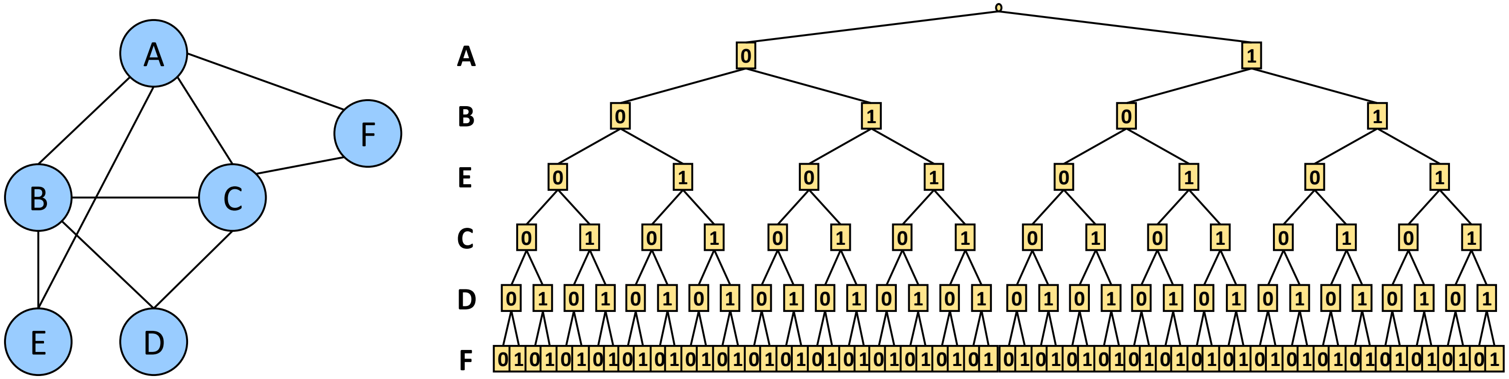


Bird's-eye View of Exact Algorithms



Inference

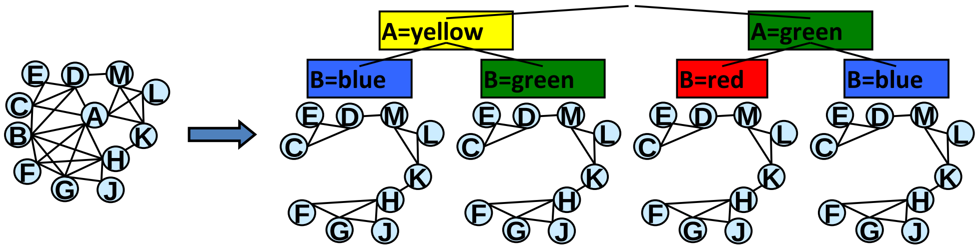
exp(w*) time/space



Search

Exp(w*) time

O(w*) space



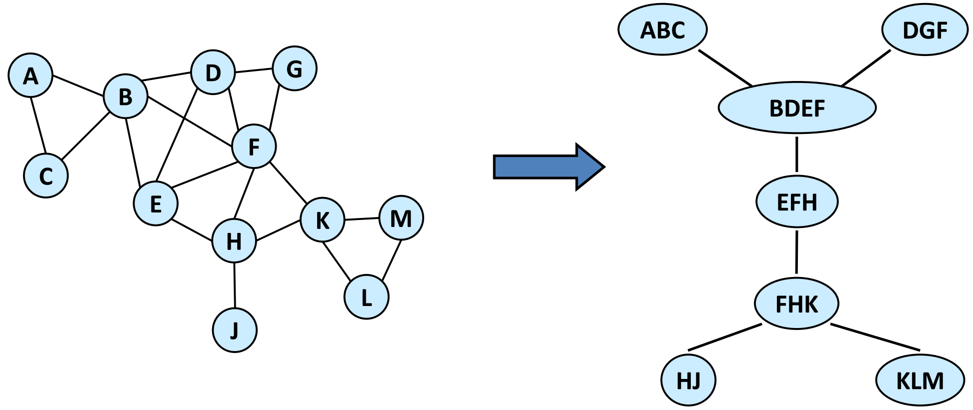
Search+inference:

Space: exp(q)

Time: exp(q+c(q))

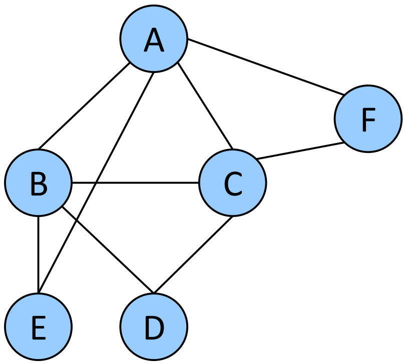
q: user controlled

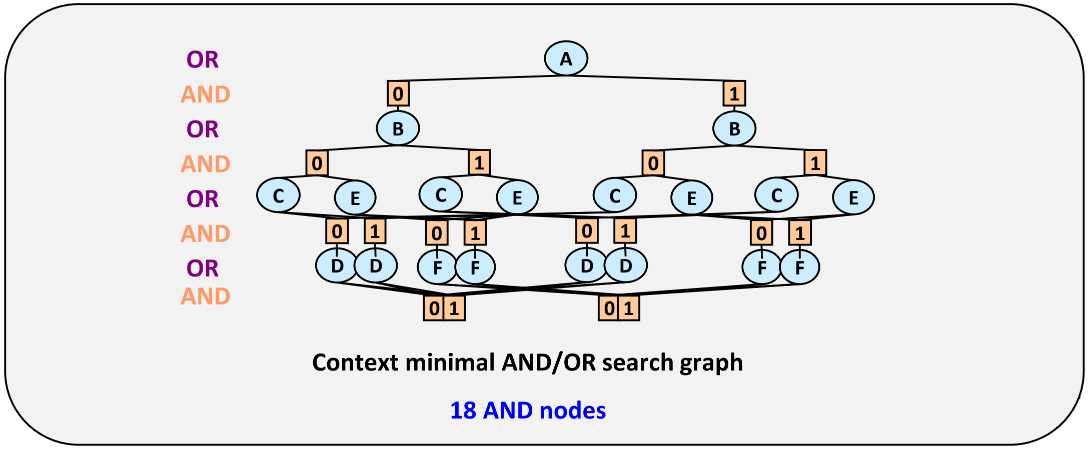
Bird's-eye View of Exact Algorithms



Inference

exp(w*) time/space





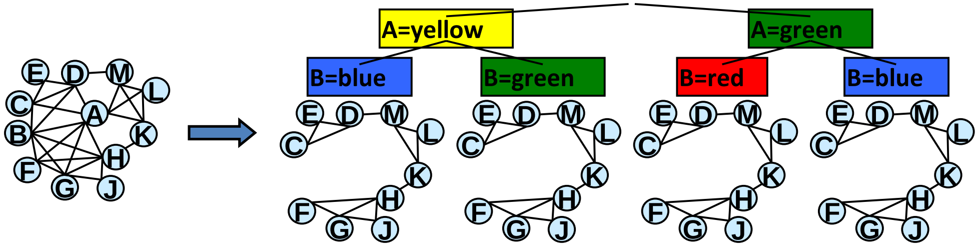
Context minimal AND/OR search graph

18 AND nodes

Search

Exp(w*) time

O(w*) space



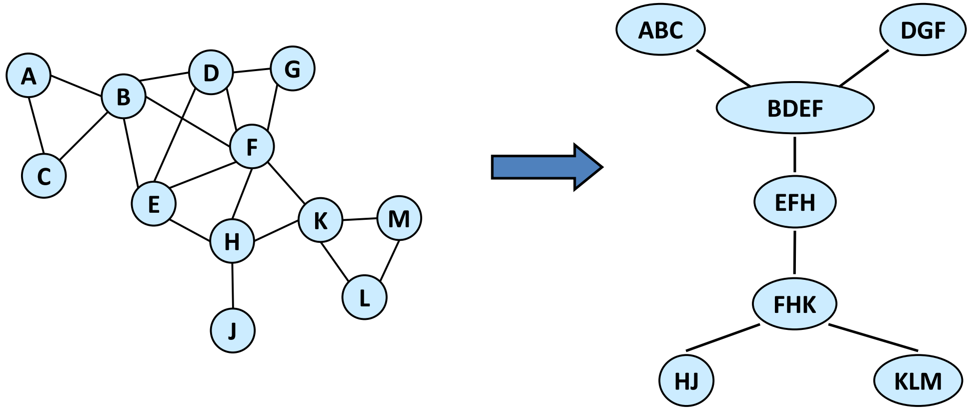
Search+inference:

Space: exp(q)

Time: exp(q+c(q))

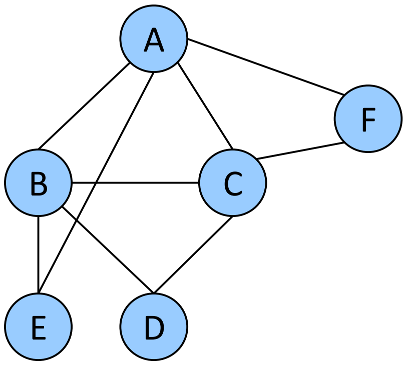
q: user controlled

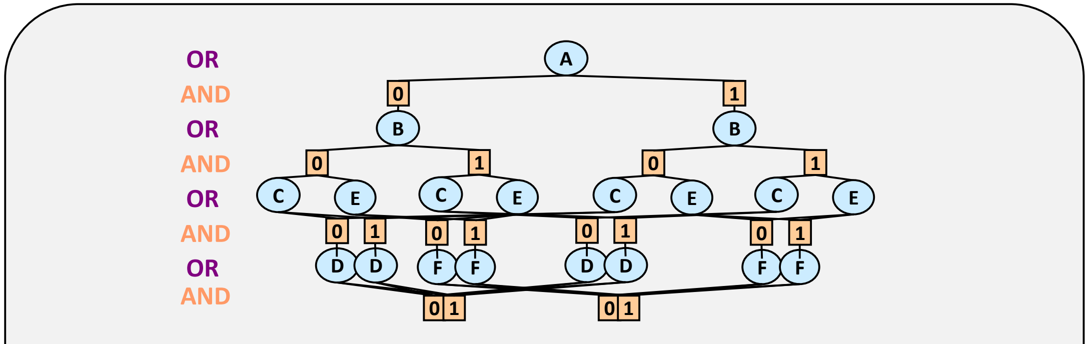
Bird's-eye View of Approximate Algorithms



Inference

Bounded Inference



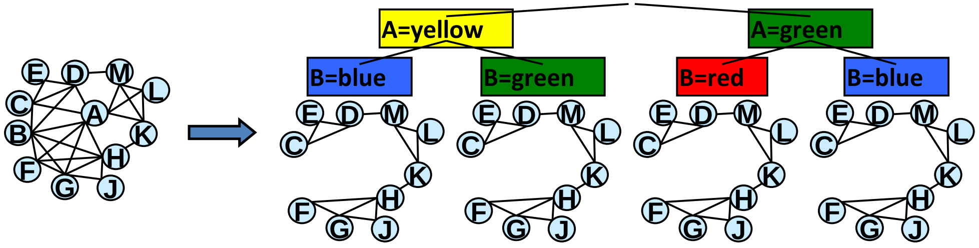


Context minimal AND/OR search graph

18 AND nodes

Search

Sampling



Search + inference:

Sampling + bounded inference

End of slides