Problem 1. Linear Models [25 points]

Consider the following (linear) causal diagram below:
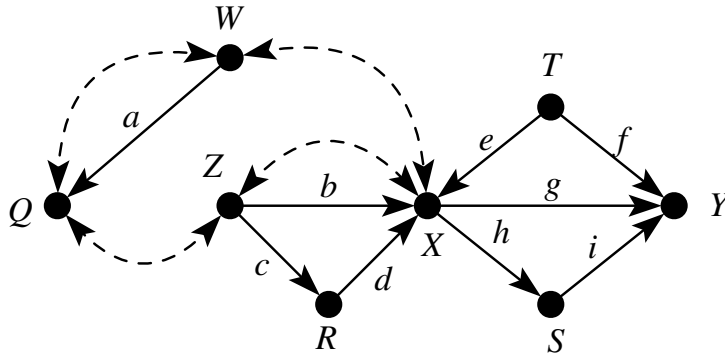
The lowercase letters next to each edge represent the corresponding structural coefficients.

(a) [5 points] Assume we perform a linear regression following the equation

$$Y = \alpha_1 X + \alpha_2 R + \alpha_3 Z + \alpha_4 Q, \tag{1}$$

where α_i, $i = 1, 2, 3, 4$ are the corresponding regression (not structural) coefficients. Is any α_i equal to 0? Explain your reasoning.

(b) [5 points] Is $E[Y|do(X)]$ identifiable in this case? If so, explain your reasoning.

(c) [15 points] Identify as many structural coefficients as possible. Justify (briefly) each answer.

Problem 2.   Study question 4.3.2 from the Primer [10 points]

(Hint: read Primer section 4.3.4)

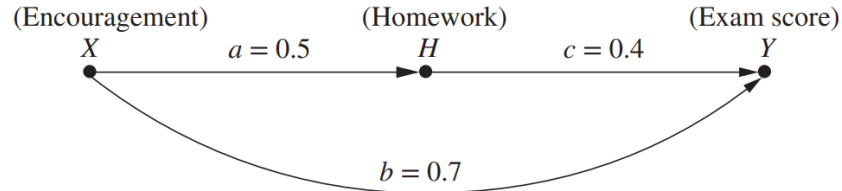(a) [5 points] Describe how the parameters a, b, c in Figure 4.1 can be estimated from nonexperimental data

(Encouragement)   (Homework)   (Exam score)
X $a = 0.5$ H $c = 0.4$ Y

$b = 0.7$

Figure 4.1   A model depicting the effect of Encouragement (X) on student's score

(b) [5 points] In the model of Figure 4.3, find the effect of education on those students whose salary is $Y = 1$.
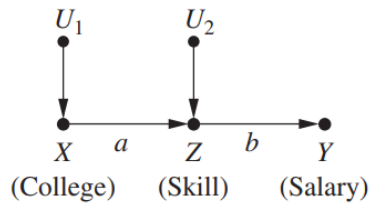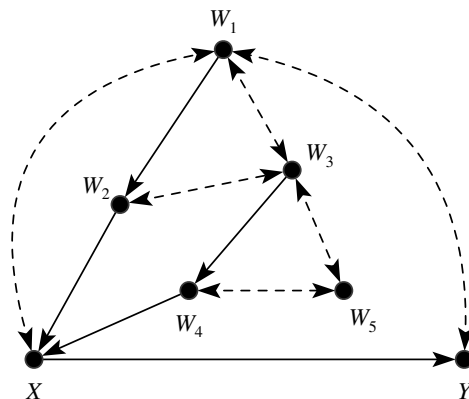[Hint: Use Theorem 4.3.2 to compute $E[Y_1 - Y_0|Y = 1]$.]

U_1 U_2

X a Z b Y
(College)   (Skill)   (Salary)

Figure 4.3   A model representing Eq. (4.7), illustrating the causal relations between college education
(X), skills (Z), and salary (Y)

Problem 3. Algorithmic Identification. [5 points]

(5 pts + 12 extra credit)

Consider the causal diagram G shown below.

(a) [5 points]
Show how $P(y \mid do(x))$ decomposes (or can be expressed) in terms of c-factors.

(b) (10 pts. extra credit) Use the `Identify` (see slides) algorithm to compute each one of the c-factors associated with the query from $P(\mathbf{v}) = Q[\mathbf{V}]$ as well as the effect $P(y \mid do(x))$. Write down the trace of the algorithm.

(c) (2 pts. extra credit) Derive the same causal effect ($P(y \mid do(x))$) using the do-calculus. (You are allowed to use the tool Fusion (url: `https://causalfusion.net`) to check your answer.)