Rina Dechter, Fall-2024
Assigned: Thursday, October 5rd
Due: Tuesday, October 15

COMPSCI 276: Causal and Probabilistic Reasoning with Graphical Models
Problem Set 1

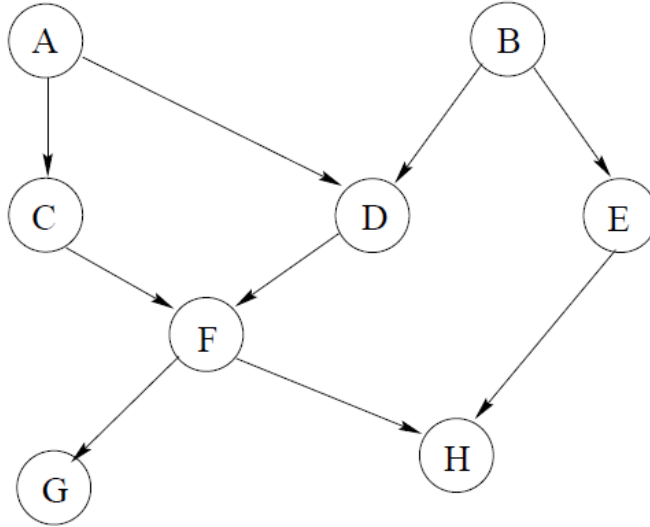Homeworks should be submitted in class and also posted on canvas.

Reading: Darwiche chapters 3 and 4.

Please indicate clearly the question numbers that you answer. It is best to type your homework. Explain your work. Do not assume I can guess what you mean!!!

1. (**20 points**, Pearl 2.1, Darwiche 3.13) There are three urns labeled one, two and three. The urns contain, respectively, three white and three black balls, four white and two black balls, and one white and two black balls. An experiment consists of selecting an urn at random, then drawing a ball from it.

   (a) (4 points) Define the set of worlds that correspond to the various outcomes of this experiments. Assume you have two variables U with values 1; 2; 3 and C with values black and white.

   (b) (4 points) Define the joint probability distribution over the set of possible worlds identified above.

   (c) (3 points) Find the probability of selecting urn 2 and drawing a black ball.

   (d) (3 points) Find the probability of drawing a black ball.

   (e) (3 points) Find the conditional probability that urn 2 was selected, given that a black ball was drawn.

   (f) (3 points) Find the probability of selecting urn 1 or a white ball.

2. (**10 points**) Use the joint-probability distribution in the table of Figure 1 to compute the following conditional probability for all values of x y and z.

   (a) $p(x|y,z)$

   (b) $p(y|x,z)$

   (c) $p(z|x,y)$

3. (**20 points**) (Darwiche, exercise 4.1) Consider the DAG (Figure 4.14):

Figure 1: Probability distributions for problem 2.

x	y	z	$p(x, y, z)$
0	0	0	0.12
0	0	1	0.18
0	1	0	0.04
0	1	1	0.16
1	0	0	0.09
1	0	1	0.21
1	1	0	0.02
1	1	1	0.18

	A	B	D	Pr(d|a, b)
	1	1	1	.5
	1	1	0	.5
	1	0	1	.6
	1	0	0	.4
	0	1	1	.1
	0	1	0	.9
	0	0	1	.8
	0	0	0	.2

A	Pr(a)
1	.2
0	.8

B	Pr(b)
1	.7
0	.3

B	E	Pr(e|b)
1	1	.1
1	0	.9
0	1	.9
0	0	.1

Figure 4.14: A Bayesian network with some of its CPTs.

(a) List the Markovian assumptions asserted by the DAG.

(b) Express $P(a, b, c, d, e, f, g, h)$ in terms of network parameters.

(c) Compute $P(A = 0, B = 0)$ and $P(E = 1|A = 1)$. Justify your answers.

(d) True or false? Explain why?

- $dsep(A, BH, E)$
- $dsep(G, D, E)$
- $dsep(AB, F, GH)$

4. **(10 points)** Consider a set of four variables $\{X, Y, Z, W\}$, which are related by:

$$I(X, \phi, Y) \text{ and } I(X, \{Y, W\}, Z).$$

Find the minimal list of independencies generated by the above two, satisfying each of the following conditions separately. (Note: the number of new independencies generated from the given ones can be zero or more.)

(a) The symmetry property.
(b) The symmetry and decomposition properties.
(c) The semigraphoid properties. (axioms 3.6a-3.6d)
(d) The graphoid properties. (axioms 3.6a-3.6e)

5. **(15 points)** Referring to the directed graph in Figure 2, determine whether or not each of the following Probabilistic independencies is true using the d-separation criterion.

(a) $I(E, \phi, G)$.
(b) $I(C, \phi, D)$.
(c) $I(C, G, D)$.
(d) $I(B, A, C)$.
(e) $I(\{C, D\}, \phi, E)$.
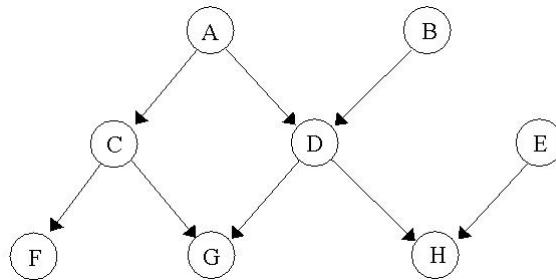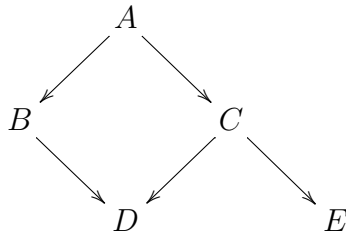(f) $I(F, A, \{E, H\})$.

Figure 2: A directed graph.

6. **(15 points)** (Question 4.14 in Darwiche book.) Suppose that the DAG

is a P-map of some distribution Pr. Construct a minimal I-map G' for Pr using each of the following variable orders:

(a) A, D, B, C, E

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(b) A, B, C, D, E

(c) E, D, C, B, A

7. **(extra credit, 10 points), Darwiche 4.24** Prove that the d-separation is equivalent to regular separation in an the ancestral graph. Namely that Z d-separates X from Y if in the moral graph that includes X, Y, Z and their ancestors Z separates X from Y.

8. **(10 points)** Using the software tool GeNie/Smile build the Bayesian network in Figure 4.4 in Darwiche book. Compute using the tool: The initial probability of each variable when there is no evidence (called beliefs). Then compute a. $P((winter|slipery\ road)$, b. $P(Winter|not\ slipery\ road)$, c. $P(sprinkler\ is\ on|not\ slippery\ road)$.