COMPSCI 276
Homework Assignment 2
Winter 2024

Instructor: Rina Dechter — Due:Wednesday, February 7th

Relevant reading: (Darwiche chapters 5, 6. Dechter chapter 4, 5).

1. (Darwiche 5.4) (15 pts) We have two sensors that are meant to detect extreme temperature, which occurs 20% of the time. The sensors have identical specifications, with a false positive rate of 1% and a false negative rate of 3%. If the power is off (dead battery), the sensors will read negative regardless of the temperature. Suppose now that we have two sensor kits: Kit A where both sensors receive power from the same battery, and Kit B where they receive power from independent batteries. Assuming that each battery has a 0.9 probability of power availability, what is the probability of extreme temperature given each of the following scenarios:

   (a) two negative sensor readings

   (b) The two sensor readings are positive?

   (c) One sensor reads positive while the other reads negative.

   Answer the previous questions with respect to each kit.

2. (30 pts) Consider the Bayesian network in Figure 1.

   (a) (20 pts) Apply BE-bel to obtain the:

      i. marginal probability of variable F.

      ii. marginal probability of variable G

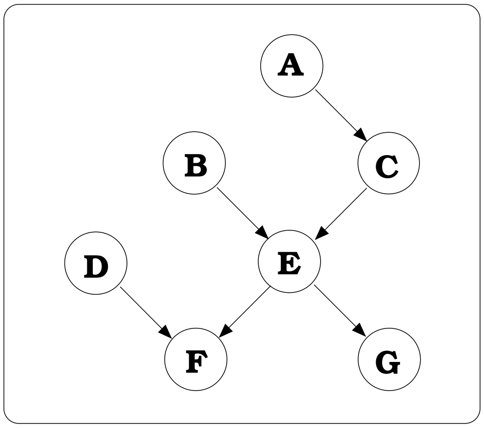
      iii. joint marginal probability of variables F and G.

      For each part show the schematic computation over buckets (you can take advantage of shared computation). The computation itself can be done manually or using any software tool of your choice.

   (b) (5 pts) Suppose now that the evidence $\{D = 0, C = 1\}$ has been observed. Apply BE-bel to obtain the probability of evidence. You can do calculation by hand or use any software tool to solve the computational parts of this question. Show the schematic computation over buckets.

   (c) (5 pts) Explain how BE-mpe can find the most probable explanation (mpe) given the evidence F=0. Demonstrate your computation. Again, you can do calculation by hand or use any software tool to solve the computational parts of this question.

3. (10 pts) (Question 6.5 in Darwiche): Consider a network N that has a single root X and n leaves $Y_1, ..., Y_n$ where N contains edges $X \rightarrow Y_i$, for all i from 1 to n.



a	$p(a)$
0	0.3
1	0.7

b	$p(b)$
0	0.6
1	0.4

d	$p(d)$
0	0.7
1	0.3

a	c	$p(c\|a)$
0	0	0.15
0	1	0.85
1	0	0.25
1	1	0.75

e	g	$p(g\|e)$
0	0	0.10
0	1	0.90
1	0	0.30
1	1	0.70

b	c	e	$p(e\|b,c)$
0	0	0	0.40
0	0	1	0.60
0	1	0	0.45
0	1	1	0.55
1	0	0	0.60
1	0	1	0.40
1	1	0	0.30
1	1	1	0.70

d	e	f	$p(f\|d,e)$
0	0	0	0.25
0	0	1	0.75
0	1	0	0.60
0	1	1	0.40
1	0	0	0.10
1	0	1	0.90
1	1	0	0.20
1	1	1	0.80

Figure 1:

(a) What is the induced-width of the order $Y_1, ..., Y_n, X$ with respect to N?

(b) What is the induced-width of the order $X, Y_1, ..., Y_n$ with respect to N?

4. (5 pts) (Question 6.8 in Darwiche) What is the induced-width of order A, B, C, D, E, F, G, H with respect to the network in Figure 2?

5. (25 pts) Given the directed graph G in Figure 4,

   (a) Compute the induced-graph along ordering: $d_1 = F, C, A, G, D, H, E, B$ and the induced-width for each variable. What is G's induced-width along d_1?

   (b) Use min-induced width (MIW) to compute an ordering, called d_2. Show its ordered graph. Compute the induced-width along d_2.

   (c) What is the induced width of the graph G? Explain your answer.

   (d) Apply BE-bel along the ordering d_1 and show the λ functions created, their placement and the expressions for deriving the functions. (optional: do the same for ordering d_2)

6. (30) Given the directed graph G in Figure 4 (used also in homework 2),

   (a) Show the bucket-tree associated with the ordering $d_1 = F, C, A, G, D, H, E, B$ and display all the messages (πs and λs) along the tree.

   (b) Assuming you performed all the computation without any evidence. How can you extract the marginal probability of D? Explain.

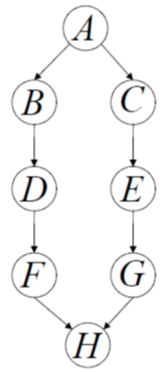


Figure 6.11: A Bayesian network structure.

Figure 2: A Bayesian network

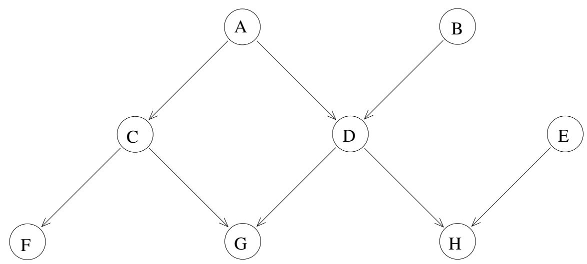


Figure 3: A directed graph

(c) Assuming you observed $F = 1$ and $B = 1$, explain how you would compute (update) $BEL(D) = P(D|F = 1, B = 1)$.

(d) Give a bound on the time and space complexity for solving this problem using O notation.

(e) Assume you have evidence over F. Describe how the loop-cutset scheme can find the belief for every variable. What is its time and space complexity?

(f) Assume you compute the beliefs using join-tree clustering. What would be the time and space complexity? Explain.

(g) Suggest an efficient scheme for solving the network without recording more than unary functions. Discuss your proposals.

7. (10) (extra credit) Let (G, P) be a Bayesian network where G is a directed acyclic graph over variables X and let $C \subseteq X$ be a subset of variables that form a loop-cutset. Prove that $P(C = c)$ can be computed in linear time and space.

8. (10) (extra credit) Which method has better time complexity, the loop-cutset method or join-tree clustering? Prove your claims.

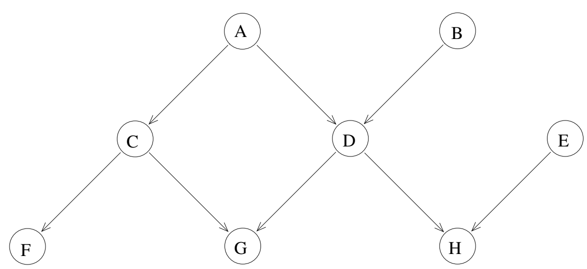


Figure 4: A directed graph