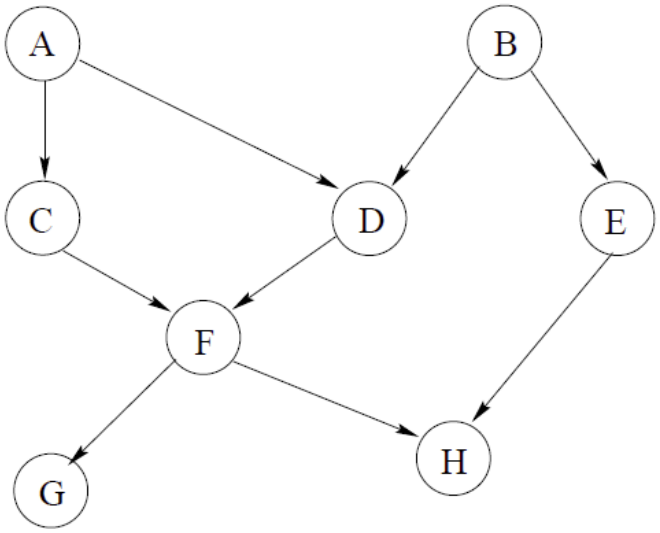
COMPSCI 276
Homework Assignment 2
Spring 2017

Instructor: Rina Dechter Due: Wednesday, April 19th

1. **(20 pt)** (Darwiche, exercise 4.1) Consider the DAG:



A	$\Pr(a)$
1	.2
0	.8

B	$\Pr(b)$
1	.7
0	.3

B	E	$\Pr(e\|b)$
1	1	.1
1	0	.9
0	1	.9
0	0	.1

A	B	D	$\Pr(d\|a,b)$
1	1	1	.5
1	1	0	.5
1	0	1	.6
1	0	0	.4
0	1	1	.1
0	1	0	.9
0	0	1	.8
0	0	0	.2

Figure 4.14: A Bayesian network with some of its CPTs.

(a) List the Markovian assumptions asserted by the DAG.

(b) Express $P(a, b, c, d, e, f, g, h)$ in terms of network parameters.

(c) Compute $P(A = 0, B = 0)$ and $P(E = 1|A = 1)$. Justify your answers.

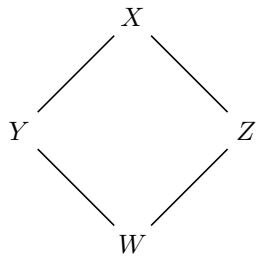
(d) True or false? Why?

- $dsep(A, BH, E)$
- $dsep(G, D, E)$
- $dsep(AB, F, GH)$

2. (**20 pt**, Pearl 3.3 a,b,d) Let $\mathbf{U} = \{X, Y, Z, W\}$, and let $P(x, y, w, z)$ be given by the following table:

X	Y	Z	W	P
1	1	1	1	1/3
1	2	2	2	1/3
2	2	1	3	1/3
all other tuples				0

(a) Show that the graph G given below is a minimal I-map of P.



(b) Show that P cannot be expressed as a product of functions on the cliques of G.

(c) Draw all the Bayesian networks of P in the orderings (X, Y, Z, W) and (W, X, Y, Z) and compute their parameters.

3. **(10 pt)** Suppose that the joint probability distribution of four variables $\{X, Y, Z, W\}$ can be factorized as:

$$p(x, y, z, w) = p(x)p(y|x)p(z|x)p(w|y, z).$$

Determine whether or not each of the following independencies holds:

(a) $I(X, Y, W)$.
(b) $I(X, Z, W)$.
(c) $I(X, \{Y, Z\}, W)$.
(d) $I(Y, \{X, W\}, Z)$.

4. **(Optional)** Consider a set of four variables $\{X, Y, Z, W\}$, which are related by:

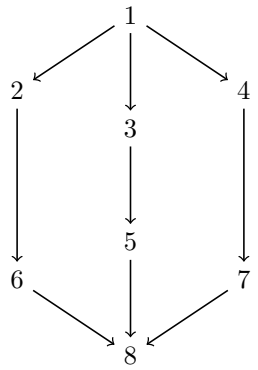
$$I(X, \phi, Y) \text{ and } I(X, \{Y, W\}, Z).$$

Find the minimal list of independencies generated by the above two, satisfying each of the following conditions separately.

(a) The symmetry property.
(b) The symmetry and decomposition properties.
(c) The semigraphoid properties. (axioms 3.6a-3.6d)
(d) The graphoid properties. (axioms 3.6a-3.6e)

5. (**10 pt**, Pearl 3.6)

   (a) (10 pt.) Find the Markov network G_0 of a probabilistic model P for which the following DAG is a perfect-map:



   (b) (extra credit, 3 pt) Find an undirected graph G such that P (in problem (a)) is decomposable relative to G.

   (c) (extra credit 3pt) Find a product form representation of P such that $P > 0$ for all events.

6. **(10 pt)** Referring to the directed graph in Figure 1, determine whether or not each of the following Probabilistic independencies is true using the D-separation criterion.

(a) $I(E, \phi, G)$.
(b) $I(C, \phi, D)$.
(c) $I(C, G, D)$.
(d) $I(B, A, C)$.
(e) $I(\{C, D\}, \phi, E)$.
(f) $I(F, A, \{E, H\})$.
(f) $I(\{A, C\}, D, \{H, E\})$.

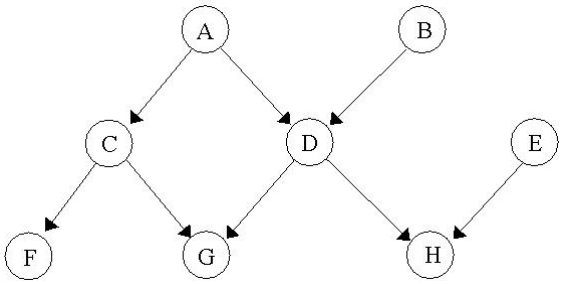
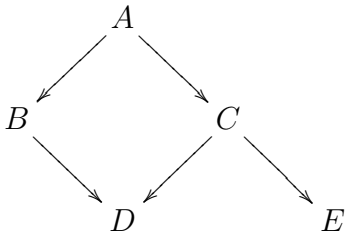


Figure 1: A directed graph.

7. **(15 pt)** (Question 4.14 in Darwiche book.) Suppose that the DAG



is a P-map of some distribution Pr. Construct a minimal I-map G' for Pr using each of the following variable orders:

   (a) A, D, B, C, E

   (b) A, B, C, D, E

   (c) E, D, C, B, A

8. **(10 pt, optional)** Given the directed graph in Figure 1, let M be the set of independencies expressed by the dag using the d-separation criterion.

   (a) Find a Bayesian network structure (a minimal I-map) of M along the ordering: F, C, D, B, A, H, E, G.

   (b) Find the Markov network of M.

9. **(extra credit, 10 pt), Darwiche 4.24** Prove that the d-separation is equivalent to regular separation in an the ancestral graph. Namely that Z d-separates X from Y if in the moral graph that includes X, Y, Z and their ancestors Z separates X from Y.