COMPSCI 276
Homework Assignment 1
Spring 2017

Instructor: Rina Dechter — Due: Monday, April 10

Reading: Darwiche chapter 3, Pearl 1-2. (Note: For probability I will use $p(.), P(.)$ or $Pr(.)$)

1. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease, and that the test is 99% accurate (i.e., the probability of testing positive given that you have the disease is 0.99, as is the probability of testing negative given that you don't have the disease). The good news is that this is a rare disease, striking only one in 10,000 people. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

2. (Darwiche, Exercise 3.2) Given the probability table on the left of Figure 1 (see next page):
   (a) What is the $Pr(A = true \vee B = true)$?
   (b) Update the distribution by conditioning on the event $A = true \vee B = true$. That is, construct the probability distribution $Pr(C|A = true \vee B = true)$.
   (c) What is the probability $Pr(A = true|A = true \vee B = true)$? What is $Pr(B = true|A = true \vee B = true)$?
   (d) Determine if the event $B = true$ is conditionally independent of $C = true$ given the event $A = true \vee B = true$

3. Use the joint-probability distribution in the table on the right of Figure 1 to compute the following conditional probability for all values of x y and z.
   (a) $p(x|y, z)$
   (b) $p(y|x, z)$
   (c) $p(z|x, y)$

4. (Pearl 2.1, Darwiche 3.13) There are three urns labeled one, two and three. The urns contain, respectively, three white and three black balls, four white and two black balls, and one white and two black balls. An experiment consists of selecting an urn at random, then drawing a ball from it.
   (a) Define the set of worlds that correspond to the various outcomes of this experiments. Assume you have two variables U with values 1; 2; 3 and C with values black and white.
   (b) Define the joint probability distribution over the set of possible worlds identified above.
   (c) Find the probability of selecting urn 2 and drawing a black ball.
   (d) Find the probability of drawing a black ball.

Figure 1: Probability distributions for problems 2 and 3.

A	B	C	$Pr(A, B, C)$
true	true	true	0.075
true	true	false	0.050
true	false	true	0.225
true	false	false	0.150
false	true	true0	0.025
false	true	false	0.100
false	false	true	0.075
false	false	false	0.300

x	y	z	$p(x, y, z)$
0	0	0	0.12
0	0	1	0.18
0	1	0	0.04
0	1	1	0.16
1	0	0	0.09
1	0	1	0.21
1	1	0	0.02
1	1	1	0.18

(e) Find the conditional probability that urn 2 was selected, given that a black ball was drawn.

(f) Find the probability of selecting urn 1 or a white ball.

5. (Darwiche 3.8) Suppose that we have a patient who was just tested for a particular disease and the test came out positive. We know that one in every thousand people has this disease. We also know that the test is not reliable: it has a false positive rate of 2 percent and a false negative rate of 5 percent.

   (a) Compute the probability that the patient has the disease given that the test was positive.

   (b) Now suppose that the test is repeated n times, and all tests come out positive. What is the smallest n for which the belief in the disease is greater than 95 percent, assuming that the errors of various tests are independent? Justify your answer.

6. (Optional, Darwiche 3.10)

7. This problem investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

   Suppose we wish to calculate $P(H|E_1, E_2)$, and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

   (a) $P(E_1, E_2), P(H), P(E_1|H), P(E_2|H)$.

   (b) $P(E_1, E_2), P(H), P(E_1, E_2|H)$.

   (c) $P(E_1|H), P(E_2|H), P(H)$.