CompSci 267 – Data Compression

- Prerequisite: CompSci 161 or 260 or 261

- Recommended textbook
  K. Sayood, *Introduction to Data Compression*,
  3rd ed., Morgan Kaufmann, San Francisco, 2006.

- Requirements
  - term project, report, and class presentation
  - weekly homework
  - no examinations

- URL `http://www.ics.uci.edu/~dan/class/267/`
  - course outline
  - homework
  - links, references

Introduction to Data Compression

- Introduction – terminology, information theory, codes
- Coding – Huffman, arithmetic
- Modeling – dictionary, context
- Text Compr'n Systems – performance
- Image Compression – lossless techniques
- Lossy Compression – quantization, coding
- Lossy Image Compr'n – JPEG, MPEG
- Audio Compression – coding, masking

Introduction

- Terminology
- Performance Measures
- Information Theory
- Codes

Terminology

- Data Compression

  transforms data to minimize size of its representation

  In contrast: Data Reliability

  is often implemented by adding check and parity bits,

  and increases redundancy and size of the data

- Motivation
  - increase capacity of storage media
  - increase communication channel capacity
  - achieve faster access to data

Applications

- Examples
  - file compression – *Gzip* (Unix), *Compactor* (Mac), *PKZIP* (PC)
  - automatic compression (disk doubler) – *Stacker*
  - facsimile transmission
  - modem compression
  - CD/DVD players
  - WWW – Flash
  - digital camera image storage – JPEG
  - HDTV
  - Teleconferencing

Applications

- Typical source formats (format depends on the application)
  - text – ASCII or EBCDIC chars (8 bits)
  - audio – real-valued fixed precision samples
  - images – pixels (picture elements)

1 bit	b/w
8 bits	grey scale
24 bits	color (3 primaries)

  - video example – an HDTV video format
    - 1920 pixels × 1080 lines × 30 fps × 8 bits/color × 3 colors → 1.5 Gbps
    - only ∼ 18 Mbps available for video
      6 MHz/channel supports only 19.2 Mbps,
      need some capacity for audio, captioning
    - requires 83:1 video compression

Applications

- What can we expect? Depends on file type, size etc.

  - Text compression (Calgary corpus) Pent-200, Win98

RK	1.81 bpc	29 kcps
LZOP	3.90 bpc	2.98 Mcps

  - Image compression

Lossless b/w JBIG	CR = 5%
Lossless grey scale JPEG	5 bpp (CR=65%)
Lossy color JPEG	0.6 bpp (CR=7%)

  - Sound compression mp3
    CD rates 1.5 Mbps, reduced to 128 Kbps: CR $\approx$ 8%

  - Video compression MPEG-1
    352$\times$240 color images @ 30 fps = 60 Mbps
    reduced to 1.2 Mbps: CR $\approx$ 2%

Terminology

- *Encoding*: compressing, reduce representation
  Decoding: recover the original data

- *Lossless*: recover precisely the original data
  Lossy: original data not recovered exactly

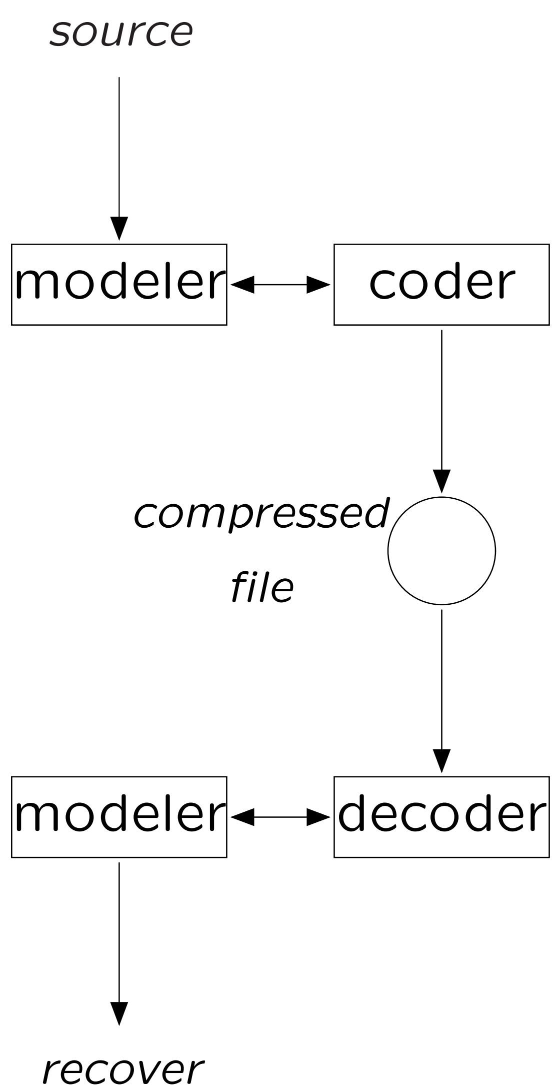
- *2-pass*: pre-scan of source required
  1-pass: no pre-scan required

- Modeling and Coding – components of the compression process

  - Modeling: describe form of redundancy
    - build abstract prototype of the source
    - select source elements for focus

  - Coding: encode model and description
    of how data differs from model, residual
    - construct new representation using model
    - map source elements to produce output

Compression-Decompression Process



Types of Source Element Redundancy

- distribution

  some elements occur more often than others,
  e.g., ‘;’ in C programs

- repetition

  elements often repeated,
  e.g., 1 or 0 in b/w image bit maps

- patterns

  correlation of symbols occurrence,
  e.g., “th, qu” in English

- positional

  some symbols occur mostly in the same relative position,
  e.g., database fields

Methods for Compression

- pre-filtering — reduce complexity of data
  may remove relevant details

- eliminate redundancy — remove any repeated information

- use human perception models — remove irrelevant detail
  in ways that minimize humans' ability to detect the information loss

- post-filtering
  attempt to further reduce/mask artifacts
  that were introduced by information loss

Performance Measures

Which measures are important depends on the application

- systemic encode/decode constraints
  - limited CPU power
  - limited working memory
  - incremental encoding
  - real-time transmittal (or multiple pass permitted)
  - real-time decode
  - random access decode enabled

  - speed – chars/sec or pixels/sec
    - Symmetric – encode + decode once
      videoconferencing, multimedia mail
    - Asymmetric – slow encode + fast multiple decode
      picture archive, video-on-demand, electronic publishing
    - Asymmetric – fast encode + slow rare decode
      file system backup, security video tapes

Performance Measures

- compression effectiveness (size reduction)
  - compression ratio CR = new file size as % of orig size
  - compression factor CF = orig file size / new file size
  - percent savings PS = amt of reduction as % of orig size
  - bit usage bpc (# bits/char), bpp (# bits/pixel)

- quality
  - fidelity – lossless, perceptually lossless, lossy

    fidelity criteria
    - MSE (mean squared error)
    - SNR (signal-to-noise ratio)
    - perceptual quality
  - allow graceful (lossy) degradation
  - allow browsing – rapid recovery of degraded version
  - delay
  - minimize error propagation

Information Theory – Information Content

- more likely events give less information
  (learn more from surprising events)
  so, measure of information content is inversely related to probability

  - n events equally likely $\Rightarrow$
    to represent each item requires $\log_2 n$ bits

- Information Content of an event having probability p
  is $\log(1/p) \quad = \quad -\log p$
  - base 2 logarithm $\rightarrow$ bits
  - base e nats, base 3 trits, base 10 hartleys

- a sequence of independent events has additive information content

Information Theory – Entropy

Shannon Entropy: a discrete memoryless source that emits n chars with probabilities $p_1, \ldots, p_n$ has entropy $H = \sum[-p_i \lg p_i]$

- entropy measures the avg # of bits needed to encode the output of a source of a random sequence
  - no compression scheme can do better
  - compression methods that approach this limit without knowledge of the pdf are called universal

- if sequence el'ts are not indep & ident distr (*iid*) then above formula gives the first-order entropy

- in physics, entropy measures disorder
  - if all n items equally likely, $H = \lg n$
  - if only 1 item can occur, $H = 0$

- entropy can be thought of as
  a measure of uncertainty as to which character is emitted next

Information Theory – Examples

Example: fair coin

$\text{Prob}(\texttt{H}) = \text{Prob}(\texttt{T}) = 1/2$

$i(\texttt{H}) = i(\texttt{T}) = 1$ bit

$H = 1$

Example: biased coin

$\text{Prob}(\texttt{H}) = 1/8$ $\quad$ $\text{Prob}(\texttt{T}) = 7/8$

$i(\texttt{H}) = 3$ bits $\quad$ $i(\texttt{T}) = 0.193$ bits

$H = .375 + .169 = 0.544$

Example: fair and biased dice

- $\text{Prob}(i) = 1/6$, $(i = 1 \ldots 6)$

  $H = 2.585$

- $\text{Prob}(1) = 3/8$, $\text{Prob}(i) = 1/8$, $(i = 2...6)$

  $H = 2.406$

- $\text{Prob}(1) = 11/16$, $\text{Prob}(i) = 1/16$, $(i = 2...6)$

  $H = 1.622$

Information Theory

- Joint Entropy of variables X, Y with joint pdf p

$$H(X,Y) \;=\; -\sum_{x\in X}\sum_{y\in Y} p(x,y)\log p(x,y)$$

- Conditional Entropy

$$H(Y|X) \;=\; -\sum_{x\in X} p(x)H(Y|X=x) \;=\; -\sum_{x\in X}\sum_{y\in Y} p(x,y)\log p(y|x)$$

  - $H(Y|X) \leq H(Y)$
  - knowing something about the context
    can reduce the uncertainty (and the entropy)
  - chain rule: $H(X,Y) \;=\; H(X) \;+\; H(Y|X)$
    Proof: $p(x,y) = p(x) * p(y|x)$
    take logs: $\log p(x,y) = \log p(x) + \log p(y|x)$
    take expectations:
    $\sum_{x,y} p(x,y)\log p(x,y) = \sum_x p(x)\log p(x) + \sum_{x,y} p(x,y)\log p(y|x)$

Information Theory

- Relative Entropy between two pdf's, p and q

$$D(p||q) = \sum_{x \in X} p(x) \log \frac{p(x)}{q(x)}$$

  - always non-negative
  - zero only if $p = q$, can be infinite
  - not symmetric and does not satisfy triangle inequality

- Mutual Information is the relative entropy between the joint pdf and the product of pdf's

$$I(X,Y) = D(\, p(x,y) || p(x)p(y)\,) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

$$= \sum_{x,y} p(x|y)p(y) \log \frac{p(x|y)}{p(x)}$$

  - $I(X,Y) = H(X) - H(X|Y) = I(Y,X)$

Information Theory – English

- information content of English
  - if 96 printable chars equally likely entropy = lg 96 = 6.6 bpc
    need $\lceil \lg 96 \rceil = 7$ bpc
  - using pdf of English text entropy ≈ 4.5 bpc
    Huffman code for this pdf = 4.7^{+} bpc
  - group text in 8-char blocks: entropy ≈ 2.4 bpc
    estimate limit for larger-size blocks = 1.3 bpc
- historical performance of best general-purpose compressors
  on Calgary corpus

Year	*bpc*	*algorithm*
1977	3.94	LZ77
1984	3.32	LZMW
1987	2.71	Gzip
1988	2.48	PPMC
1994	2.33	PPMD
1995	2.29	BWT
1997	1.99	BOA
1999	1.82	RK
2009	1.77	ZPAQ slow, much memory (unconfirmed)
2009	1.51	PAQ8 variant, very slow, very much memory (unconfirmed)

Codes

- Types of codes
- Fixed finite codes
- Fixed infinite codes

Types of Codes

classified by time variance of codewords

A code is a mapping
from source = string over alphabet S
to compressor output = stream of codewords over alphabet C

- Fixed code
  codeword set is time invariant
  selection is predetermined

- Static code
  codeword set is time invariant
  selection dictated by model

- Adaptive code
  dynamic codeword set (varies with time)
  selection/modification dictated by model

Types of Codes

classified by input-output rates

(for time-invariant codes, think of parse+codeword lengths)

- Fixed-to-Fixed rate code: $S \to C$

  ASCII code

- Fixed-to-Variable rate code: $S \to C^+$

  Morse, Huffman codes

- Variable-to-Fixed rate code: $S^+ \to C$

  also called *free-parse* methods

  Tunstall code, Lempel-Ziv methods

- Variable-to-Variable rate code: $S^+ \to C^+$

  Runlength encoding, Arithmetic coding

Types of Codes
classified by decodability

- **Ambiguous Code** — ∃ 2 strings with identical encoding

symbol	a	b	c	d
code	1	01	010	001

  - dilemma: 01001 could be bd or cb

- **Complete** — every semi-infinite string is decodable

- **Uniquely Decodable** — unambiguous and complete

- **Instantaneous** — can be decoded as codewords are received
  Prefix Code — no codeword is a prefix of any other
  - C is a prefix code $\Leftrightarrow$ C is instantaneous

    Example:

symbol	a	b
code	0	01

    - *not* complete, as 110... is undecodable
    - unambiguous but not instantaneous

Fixed Codes

- Advantages – agreed upon beforehand
  - no need for encoder to transmit code
  - faster because selecting, not computing code

- Disadvantages – does not exploit info from model
  - code unrelated to particular text
  - can't remove inherent data redundancy

- One fixed length provides no compression
  Example: ASCII, EBCDIC
  - modeler does all the work, packing information into 'events'

- Multiple fixed-length (*e.g.*, 6 and 12) gain compression by using
  - shorter codes for expected freq chars
  - longer codes for infrequent chars

  Example: Morse

Fixed Finite Codes – Enumerative Coding

- for a known finite set S of n elements (chars)
  map S to the set $\{0, 1, \ldots, n-1\}$ (can refer to elements via indices)
  - problem is reduced to representing integers $0, 1, 2, \ldots, n-1$
    which normally requires $\lceil \lg n \rceil$ bits

- to decode requires inverse map, can implement by
  - maintaining a table, or
  - use algorithm to compute maps

- no compression if all codewords are same length
  var-length code gains some compression

Enumerative Coding – Example

$S = \{$ length-m binary strings having exactly j ones $\}$

- $n = |S| = \binom{m}{j} = \frac{m!}{j!(m-j)!}$
- can compute index $I \in [0, n-1]$ for $X = x_{m-1} \ldots x_1 x_0$
  let positions $\{p_i\}$ in X that contain ones be
  $$m - 1 \geq p_1 > p_2 > \cdots > p_j \geq 0$$
  index $I = \binom{p_1}{j} + \binom{p_2}{j-1} + \binom{p_3}{j-2} + \cdots + \binom{p_j}{1}$
- can compute inverse map: index $I \rightarrow$ string

$r \leftarrow I$

for $i \leftarrow 1$ **to** j

$\quad p_i \leftarrow$ max t s.t. $\binom{t}{j+1-i} \leq r$

$\quad r \leftarrow r - \binom{p_i}{j+1-i}$

Enumerative Coding – Example

Example: $m = 6, \quad j = 4 \rightarrow n = 15$

- compute index(110110)
  - 1-bits in positions 5,4,2,1
  - index $= \binom{5}{4} + \binom{4}{3} + \binom{2}{2} + \binom{1}{1} = 11$

- compute inverse of index 9
  - maximize $\binom{t}{4} \leq 9 \qquad\qquad \rightarrow p_1 = 5$
  - maximize $\binom{t}{3} \leq 9 - \binom{5}{4} = 4 \rightarrow p_2 = 4$
  - maximize $\binom{t}{2} \leq 4 - \binom{4}{3} = 0 \rightarrow p_3 = 1$
  - maximize $\binom{t}{1} \leq 0 - \binom{1}{2} = 0 \rightarrow p_4 = 0$

  $\Rightarrow$ sequence 110011

Fixed Finite Codes – Phasing-In

To represent n codes normally need $B = \lceil \lg n \rceil$ bits

If n not power of 2, can sometimes use $B - 1$ bits

- $i < 2^B - n \;\Rightarrow\;$ encode i ($B - 1$ bits)
- $i \geq 2^B - n \;\Rightarrow\;$ encode $i + 2^B - n$ (B bits)
- save some space
  increase encode/decode time

Example: $n = 5 \;\Rightarrow\; B = 3$

i	*code*
0	00
1	01
2	10
3	110
4	111

Fixed Finite Codes – Start-Step-Stop Codes

family based on choice of 3 parameters

- k codeword sets, $k = (stop - start)/step + 1$
- set n has codewords = `111...1 0 xxx...x`

  prefix: $(n-1)$ 1's, one 0 (omit when $n = k$)

  suffix: $start + (n-1) * step$ bits

Example: $start = 3, step = 2, stop = 9$

n=1	`0xxx`	{`0000`, `0001`, `0010`, `0011`, `0100`, ... `0111`}
n=2	`10xxxxx`	32 codewords
n=3	`110xxxxxxx`	128 codewords
n=4	`111xxxxxxxxx`	512 codewords, 680 in all

- instantaneously decodable:

  read n 1's until either 0 encountered

  or $n = (stop - start)/step$

  read $start + (n-1) * step$ more bits, build #

Example: $start = 3, \ step = 2, \ stop = 9$

1	1	0	1	1	0	1	0	1	1
8	+32	(n=3)	+64	+32		+8		+2	+1

Fixed Infinite Codes

- for a set of unknown or increasing size
  encode 1,2,3,...
  with codewords of increasing length

- popular codes
  - Elias
  - Even-Rodeh
  - Zeckendorf (Fibonacci)
  - Golomb and Rice
  - variable-byte

Elias Gamma Code

- instantaneously decodable:

  read n 0-bits until encounter 1 bit (starts X)

  read n more bits, computing binary value X

Example:

```
0   0   0    1   0   0   1
1   2   3=n  8   8   8   9
```

- to encode j: $\lfloor \lg j \rfloor$ 0's

  followed by binary value of j

- length of encoding$(j) = 2\lfloor \lg j \rfloor + 1$

Gamma Code	*Integer*	*Bits*
1	1	1
01x	2 - 3	3
001xx	4 - 7	5
0001xxx	8 - 15	7
00001xxxx	16 - 31	9
000001xxxxx	32 - 63	11

Elias Delta Code

- instantaneously decodable:

  read an Elias Gamma Code number V

  read $V-1$ more bits, computing binary value W

  $X = 2^{V-1} + W$

Example:

0	0	1	0	0	0	0	1
1	2=n	4	4	4=V	8	8	9

- to encode j: $\lfloor \lg(1 + \lg j) \rfloor$ 0's
  then binary value of $1 + \lfloor \lg j \rfloor$
  then binary value of $j - 2^{\lfloor \lg j \rfloor}$, using $\lfloor \lg j \rfloor - 1$ bits

- length of encoding of $j \approx \lg j + 2 \lg \lg j$

Delta Code	*Integer*	*Bits*
1	1	1
010x	2- 3	4
011xx	4- 7	5
00100xxx	8-15	8
00101xxxx	16-31	9
00110xxxxx	32-63	10

Elias Omega Code

- instantaneously decodable:

  $j \leftarrow 0$

  while peek(next bit)=1 **do**

  $j \leftarrow j + 1$

  $j \leftarrow$ compute value of next j bits

- to encode j: write 0

  while $j \geq 2$

  prepend binary value of j

  $j \leftarrow \lfloor \lg j \rfloor$

- each group encodes len(next group)-1

  the first group has length 2

Omega Code	*Integer*	*Bits*
0	1	1
1x0	2- 3	3
101xx0	4- 7	6
111xxx0	8-15	7
101001xxxx0	16-31	11
101011xxxxx0	32-63	12

Even-Rodeh Code

- similar to Elias Omega

- to encode $j \le 3$: express as 3 bits (has leading 0)
- to encode $j \ge 4$: write 0
  while $j \ge 8$
  prepend binary value of j
  $j \leftarrow \lfloor \lg j \rfloor + 1$

- each group encodes len(next group)
  the first group has length 3

Even-Rodeh	*Integer*	*Bits*
0xx	0- 3	3
1xx0	4- 7	4
1001xxx0	8-15	8
1011xxxx0	16-31	9
1101xxxxx0	32-63	10

- compare

value	1	2	4	8	16	32	64	128	256
Elias ω	1	3	6	7	11	12	13	14	21
Even-Rodeh	3	3	4	8	9	10	11	17	18

Zeckendorf (Fibonacci) Code

- to encode j:
  - express j as sum of Fibonacci #'s
  - represent as bit vector (1,2,3,5,8,13,...)
  - forbidden to have two adjacent 1's
  - append a 1-bit

- instantaneously decodable:
  - 11 delimits end of codeword
  - while parsing, build value based on Fib #'s

Example:

0	1	0	1	0	0	1	1
1	2	3	5	8	13	21	end

- length of encoding of $j = \lfloor \log_\phi j \rfloor + 1$

- Advantages
  - for most pdf's, Fib outperforms Elias
  - robust: 1-bit errors disturb ≤ 3 chars

Compare Elias & Fibonacci Code Lengths

Integer	*Gamma*	*Delta*	*Fibonacci*
1	1	1	2
2	3	4	3
3	3	4	4
4	5	5	4
5-7	5	5	5
8-15	7	8	6-7
16-31	9	9	7-8
32-63	11	10	8-10
64-88	13	11	10
100	13	11	11
1000	19	16	16
10^4	27	20	20
10^5	33	25	25
10^6	39	28	30

Asymptotically, Elias Delta uses fewer bits but,
if most numbers are small (< 7) and others not too large
then Elias Gamma is best

Golomb Codes

- family of codes, with one parameter (m)
  - start with code for 0
  - designed for coding asymmetric

    binary events with probs $p >> (1 - p)$

    by encoding runlengths of probable event
  - $p^m = 0.5 \rightarrow \text{Pr}(\text{len } n + m) = \frac{1}{2}\text{Pr}(\text{len } n)$

    so codeword($n + m$) should have

    one more bit than for n

- to encode j
  - transmit unary value of $\lfloor j/m \rfloor$

    uses $\lfloor j/m \rfloor$ 1-bits and one 0-bit
  - transmit phased-in binary code for $j \bmod m$

    uses $\lfloor \lg m \rfloor$ or $\lceil \lg m \rceil$ bits

Golomb Codes

Integer	$m = 3$	$m = 4$	$m = 5$	$m = 6$
0	00	000	000	000
1	010	001	001	001
2	011	010	010	0100
3	100	011	0110	0101
4	1010	1000	0111	0110
5	1011	1001	1000	0111
6	1100	1010	1001	1000
7	11010	1011	1010	1001
8	11011	11000	10110	10100

Rice Codes

- family of codes, with one parameter (k)
  special case of Golomb codes, $m = 2^k$
- to encode j transmit $\lfloor j/2^k \rfloor$ 1-bits, then one 0-bit
  transmit k least significant bits of j
- length of encoding of $j = \lfloor j/2^k \rfloor + k + 1$

k = 2:	`0xx`	0-3	k = 3:	`0xxx`	0-7
	`10xx`	4-7		`10xxx`	8-15
	`110xx`	8-11		`110xxx`	16-31
	`1111111001`	33		`11110001`	33

- instantaneously decodable
  position of 0 gives value prefix
  append next k bits

Variable-Byte Code

- simple binary, using minimum required number of bytes
- each byte: value (7 bits), is this last byte? (1 bit)

Integer	*Bits*
0 - 127	8
128 - 16383	16
16384 - 2097151	24

- integral byte sizes for easy coding
- effective for medium-size numbers
- wasteful for small numbers

Comparing Fixed Codes