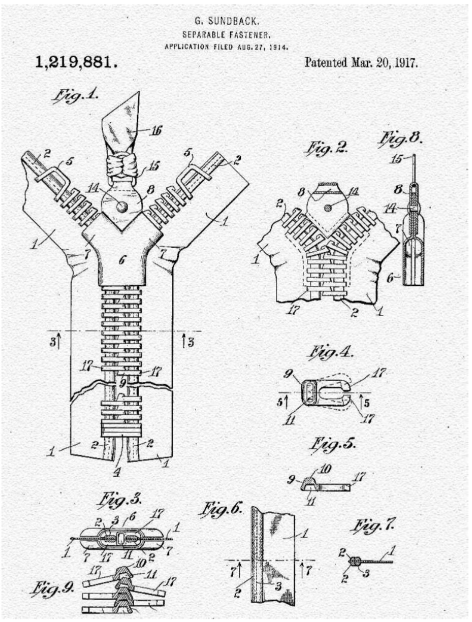
Presentation for use with the textbook, Algorithm Design and Applications, by M. T. Goodrich and R. Tamassia, Wiley, 2015

Merge Sort



Separable Fastener, U.S. Patent 1,219,881, 1917.
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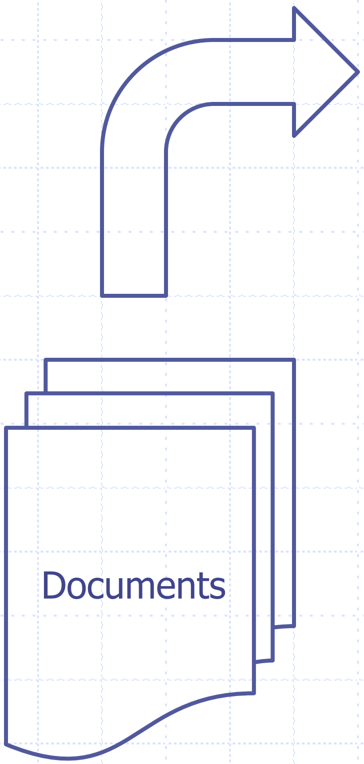
Application: Internet Search Engines

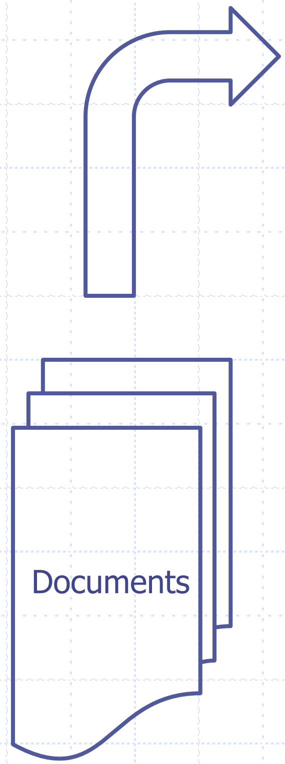
- Sorting has a lot of applications, including uses in Internet search engines.
- Sorting arises in the steps needed to build a data structure, known as the **inverted file** or **inverted index**, that allows a search engine to quickly return a list of the documents that contain a given keyword.



Word	Document Number & word location
banana	1:3, 2:45
butterfly	2:15, 3:12
camel	4:40
dog	1:60, 1:70, 2:22, 3:20, 4:11
horse	4:21
pig	2:55
pizza	1:56, 3:33

Application: How Sorting Builds an Internet Search Engine

- To build an inverted file we need to identify, for each keyword, k, the documents containing k.
- Bringing all such documents together can be done simply by sorting the set of keyword-document pairs by keywords.
- This places all the (k, d) pairs with the same keyword, k, right next to one another.
- From this sorted list, it is then a simple computation to scan the list and build a lookup table of documents for each keyword that appears in this sorted list.



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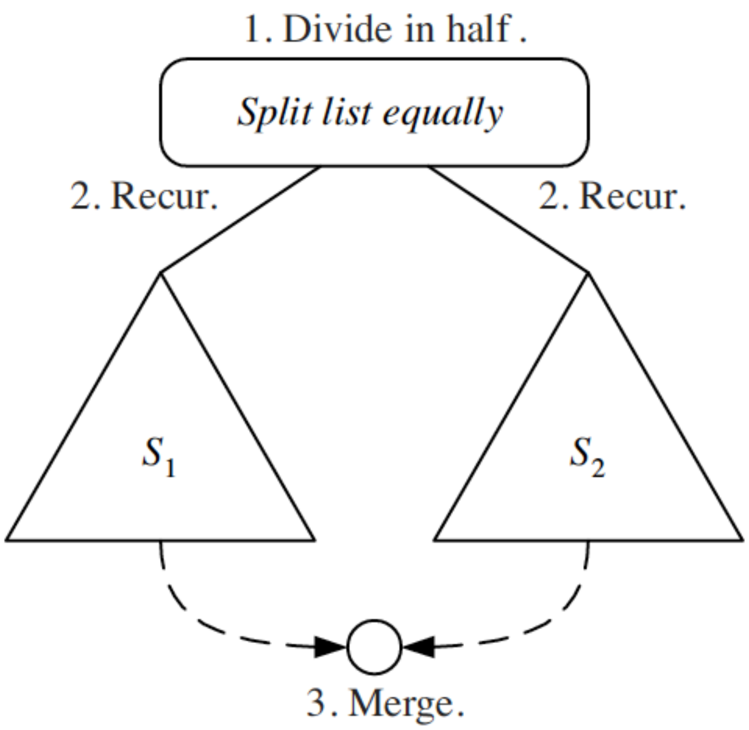
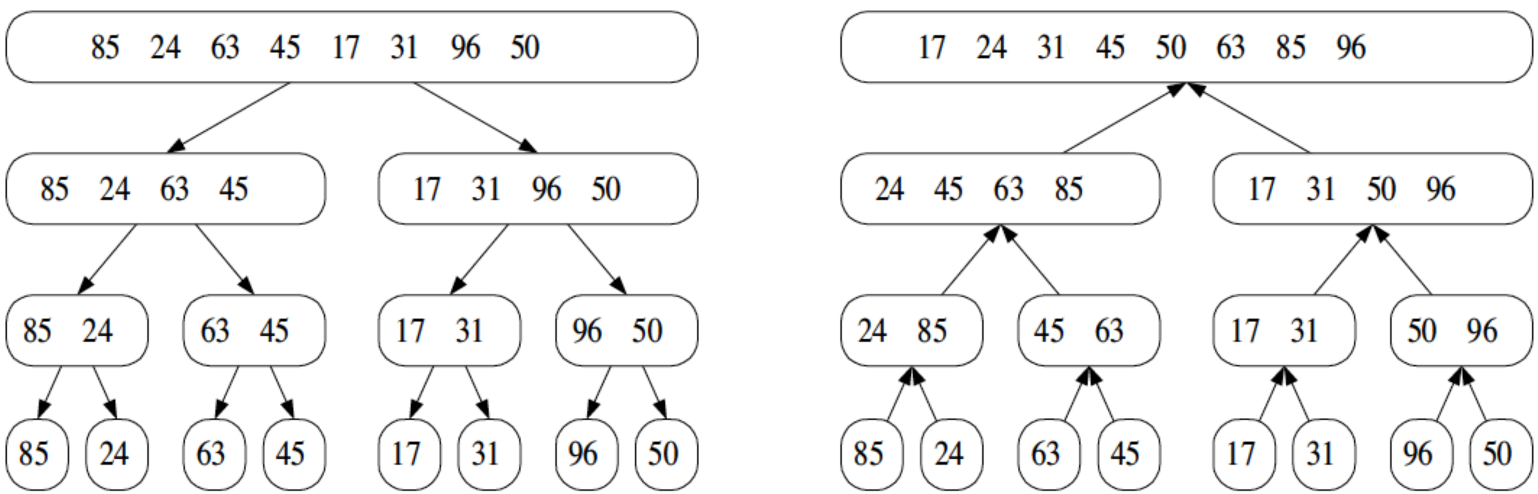
Divide-and-Conquer

- Divide-and conquer is a general algorithm design paradigm:
  - Divide: divide the input data S in two disjoint subsets S_1 and S_2
  - Recur: solve the subproblems associated with S_1 and S_2
  - Conquer: combine the solutions for S_1 and S_2 into a solution for S
- The base case for the recursion are subproblems of size 0 or 1



Merge-Sort

- Merge-sort is a sorting algorithm based on the divide-and-conquer paradigm
- Like heap-sort
  - It has $O(n \log n)$ running time
- Unlike heap-sort
  - It does not use an auxiliary priority queue
  - It accesses data in a sequential manner (suitable to sort data on a disk)



The Merge-Sort Algorithm

- Merge-sort on an input sequence S with n elements consists of three steps:
  - Divide: partition S into two sequences S_1 and S_2 of about $n/2$ elements each
  - Recur: recursively sort S_1 and S_2
  - Conquer: merge S_1 and S_2 into a unique sorted sequence

Algorithm $\textit{mergeSort}(S)$
    Input sequence S with n elements
    Output sequence S sorted according to C
    if $S.\textit{size}() > 1$
        $(S_1, S_2) \leftarrow \textit{partition}(S, n/2)$
        $\textit{mergeSort}(S_1)$
        $\textit{mergeSort}(S_2)$
        $S \leftarrow \textit{merge}(S_1, S_2)$

Merging Two Sorted Sequences

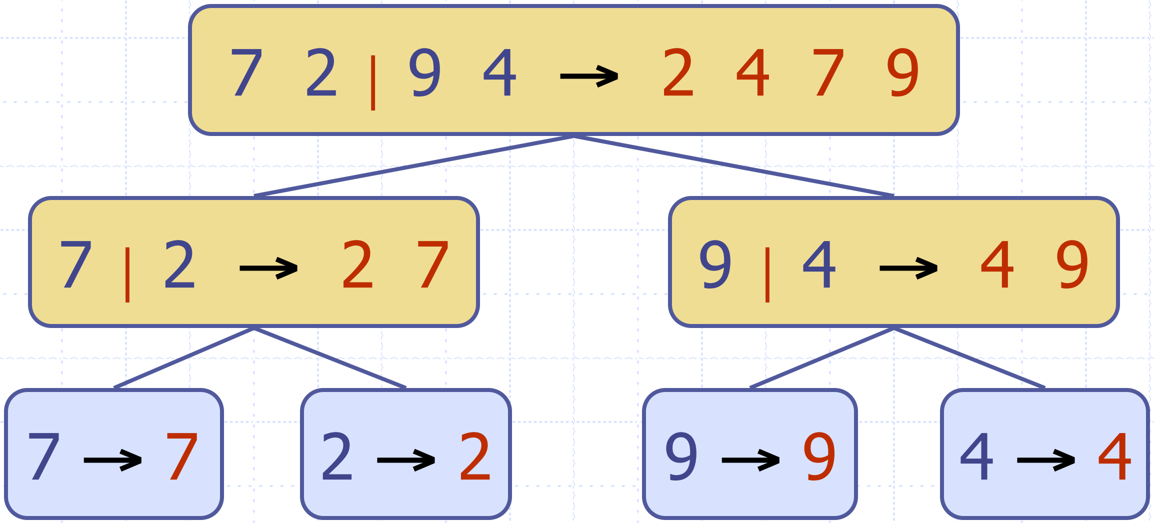
- The conquer step of merge-sort consists of merging two sorted sequences A and B into a sorted sequence S containing the union of the elements of A and B
- Merging two sorted sequences, each with $n/2$ elements and implemented by means of a doubly linked list, takes $O(n)$ time

Algorithm merge(S_1, S_2, S):
    Input: Two arrays, S_1 and S_2, of size n_1 and n_2, respectively, sorted in non-decreasing order, and an empty array, S, of size at least $n_1 + n_2$
    Output: S, containing the elements from S_1 and S_2 in sorted order

```
i ← 1
j ← 1
while i ≤ n and j ≤ n do
    if S1[i] ≤ S2[j] then
        S[i + j − 1] ← S1[i]
        i ← i + 1
    else
        S[i + j − 1] ← S2[j]
        j ← j + 1
while i ≤ n do
    S[i + j − 1] ← S1[i]
    i ← i + 1
while j ≤ n do
    S[i + j − 1] ← S2[j]
    j ← j + 1
```

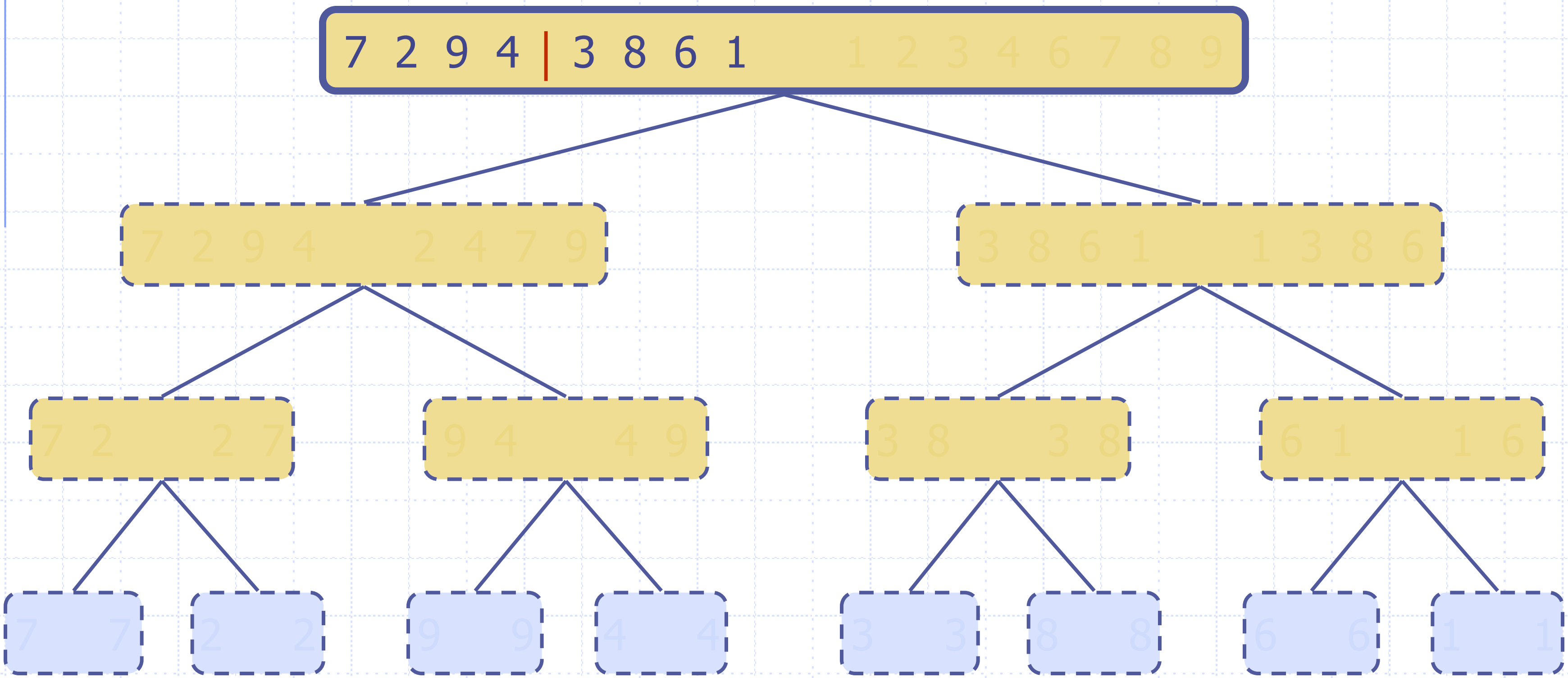
Merge-Sort Tree

- An execution of merge-sort is depicted by a binary tree
  - each node represents a recursive call of merge-sort and stores
    - unsorted sequence before the execution and its partition
    - sorted sequence at the end of the execution
  - the root is the initial call
  - the leaves are calls on subsequences of size 0 or 1



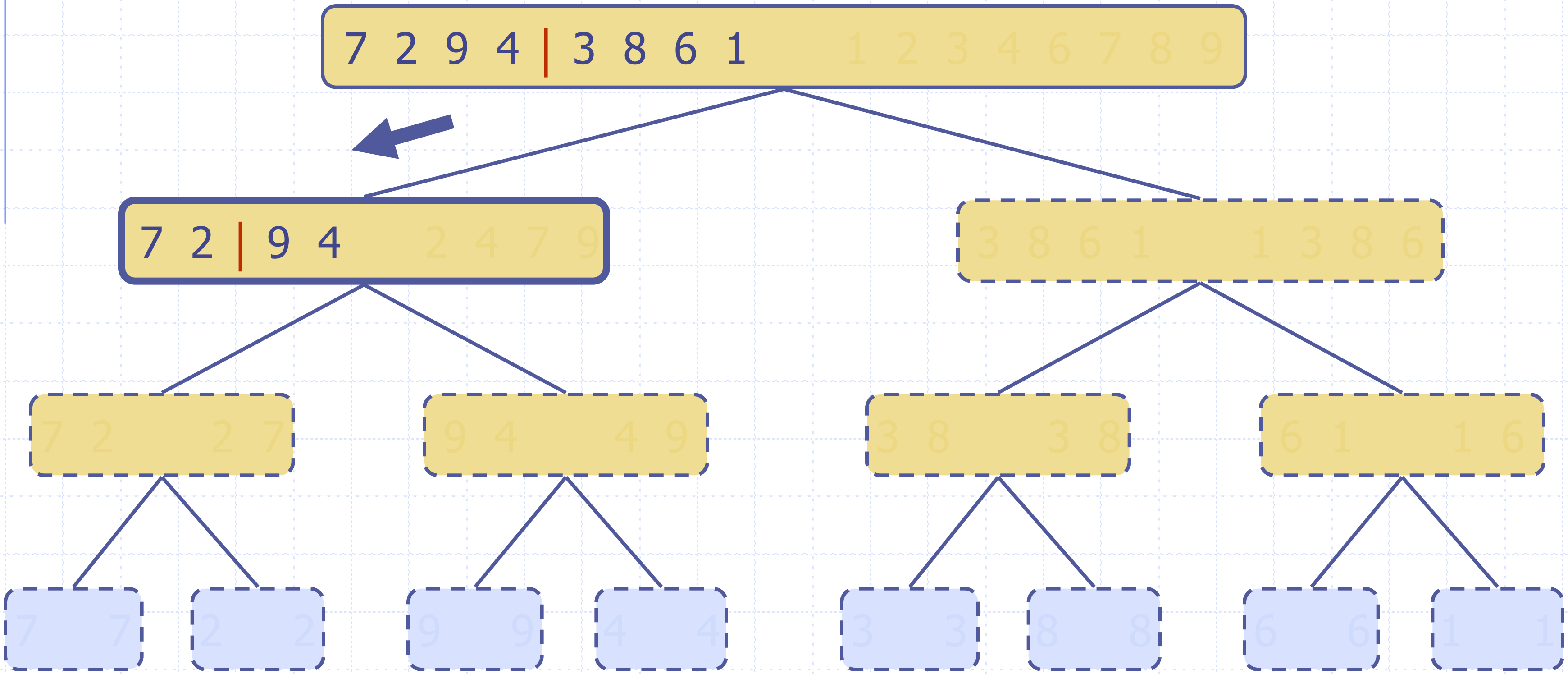
Execution Example

- Partition



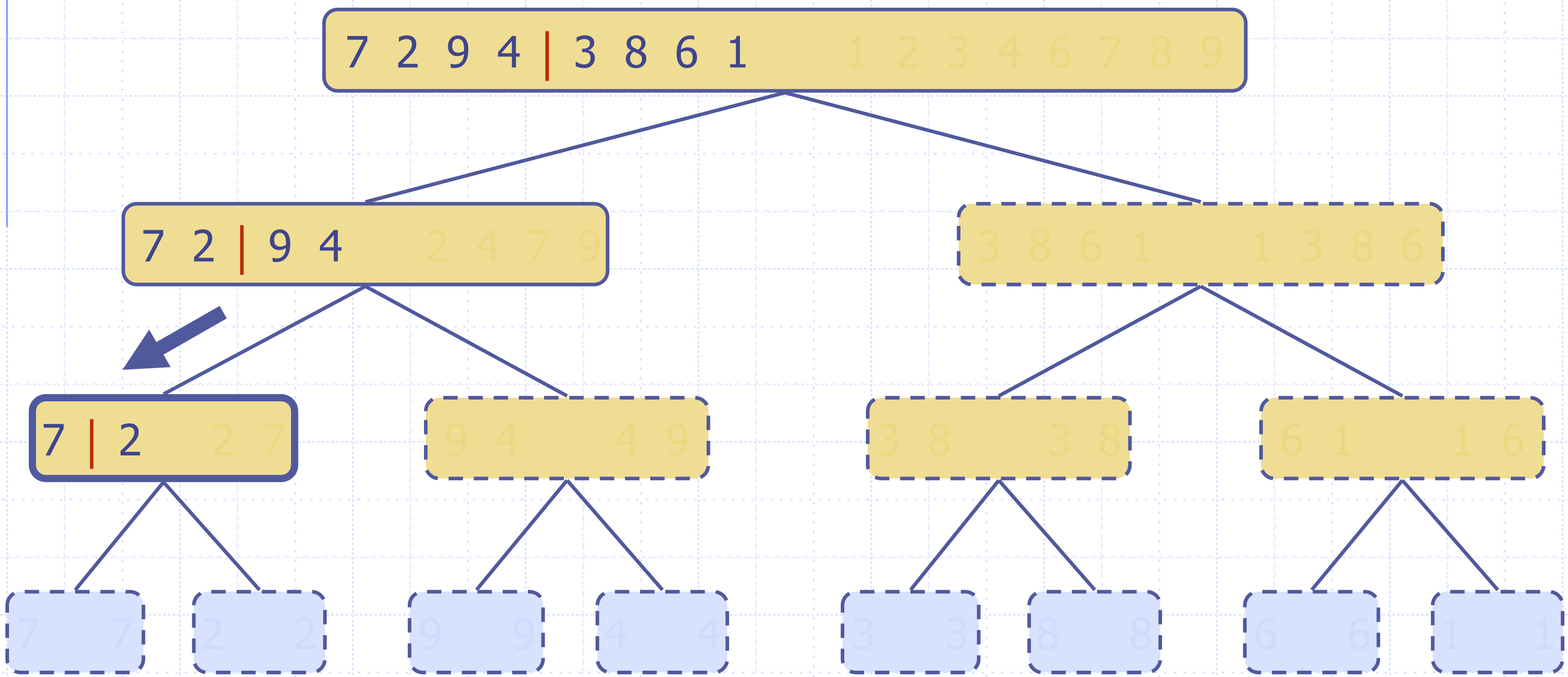
Execution Example (cont.)

- Recursive call, partition



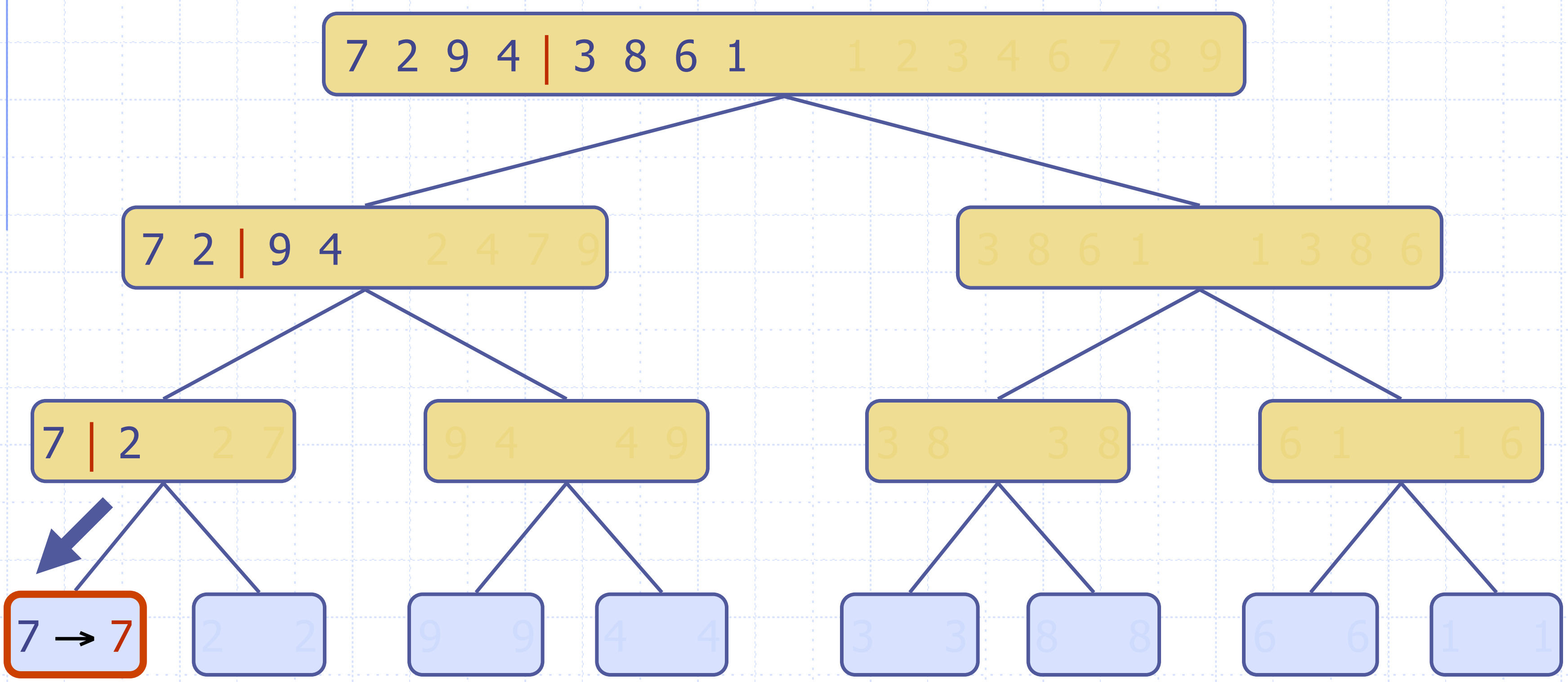
Execution Example (cont.)

- Recursive call, partition



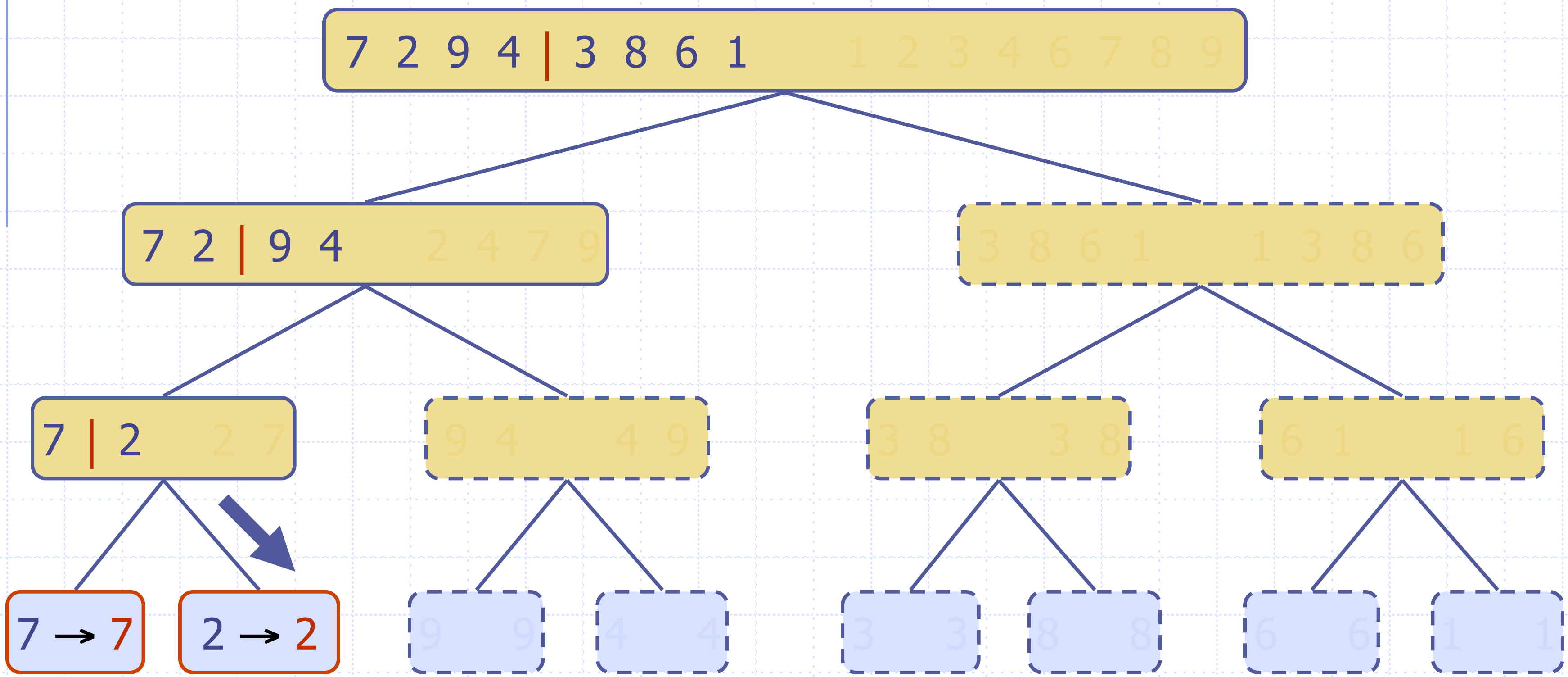
Execution Example (cont.)

- Recursive call, base case



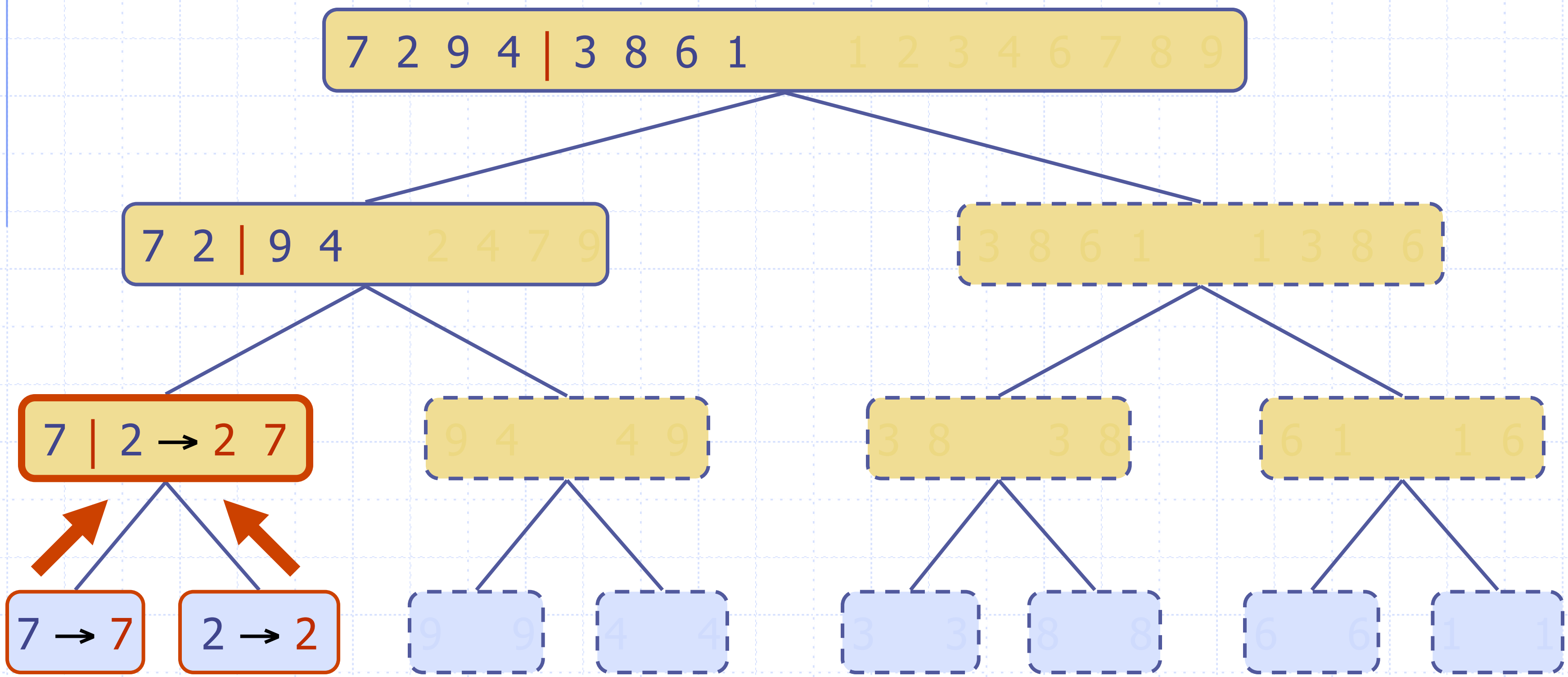
Execution Example (cont.)

- Recursive call, base case



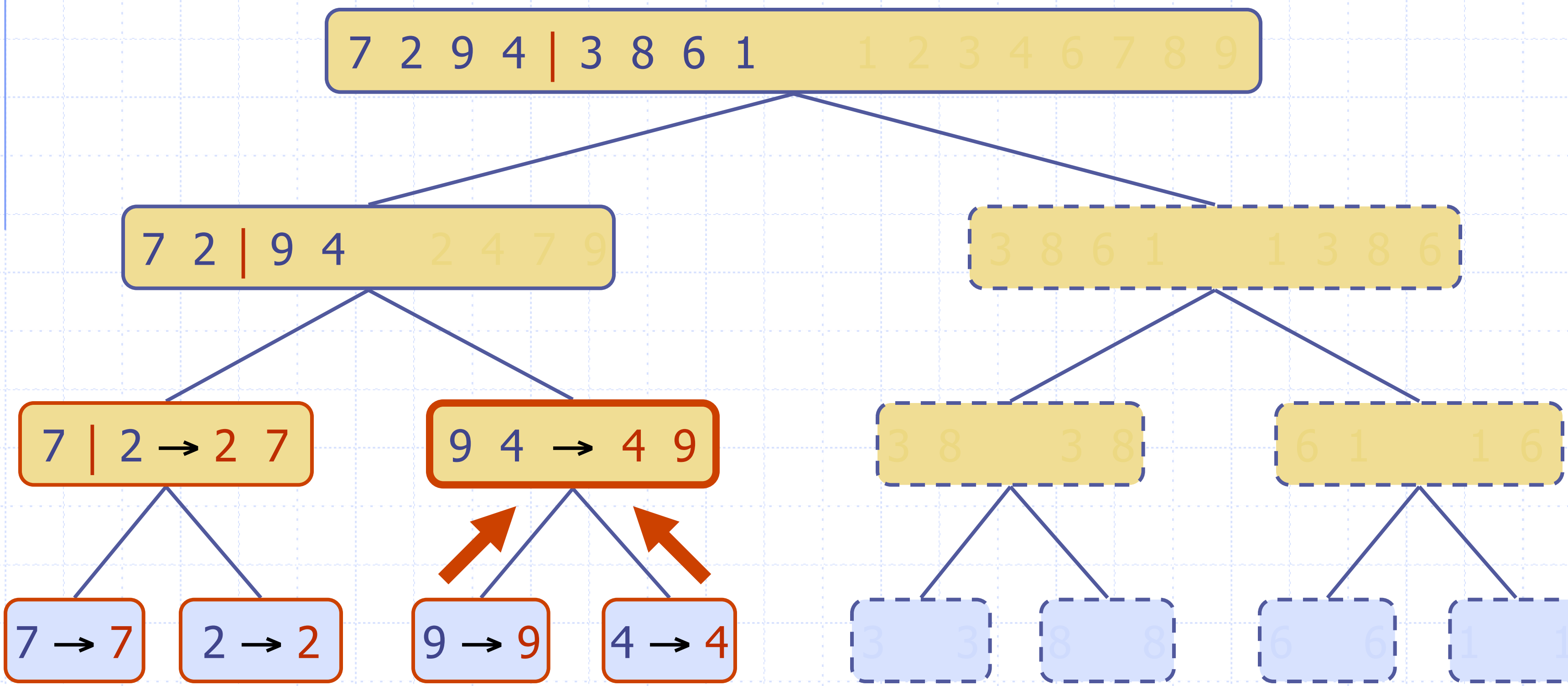
Execution Example (cont.)

- Merge



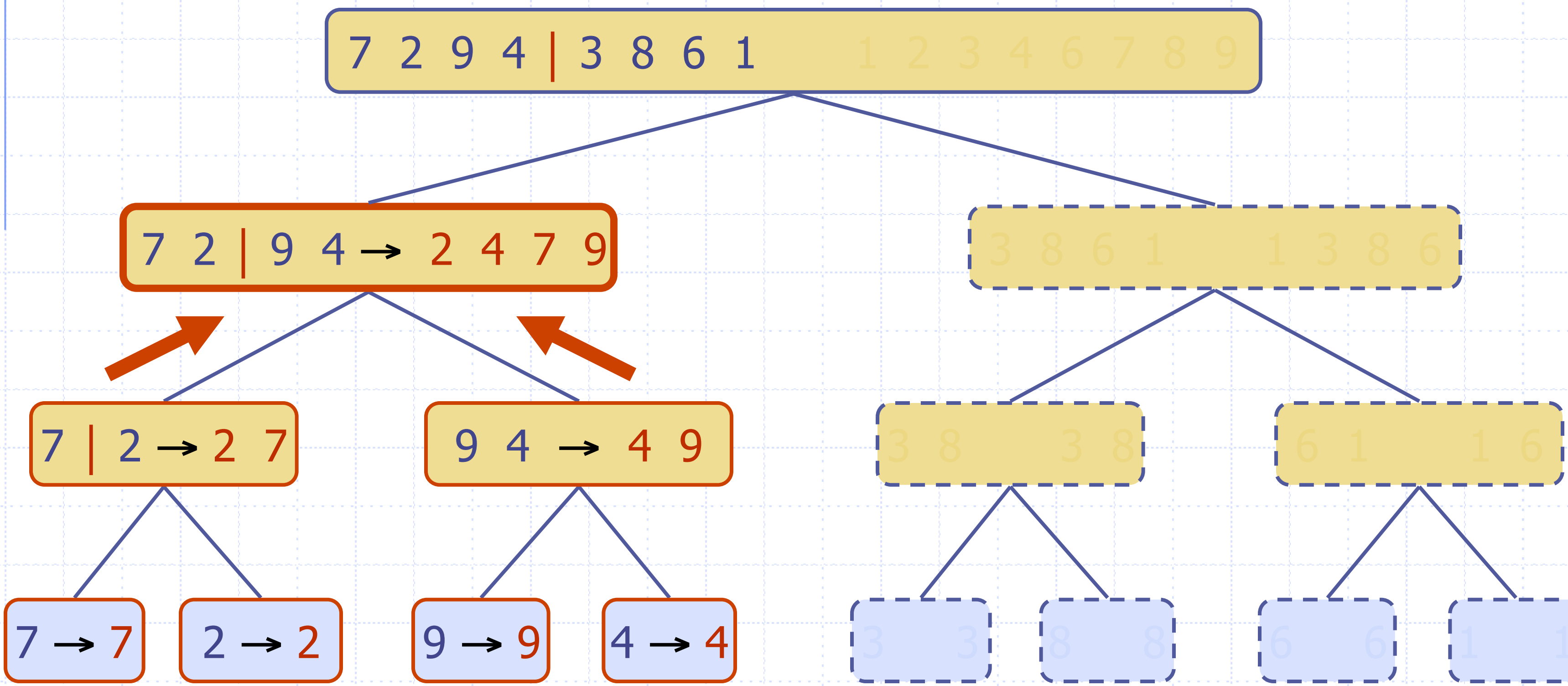
Execution Example (cont.)

- Recursive call, …, base case, merge



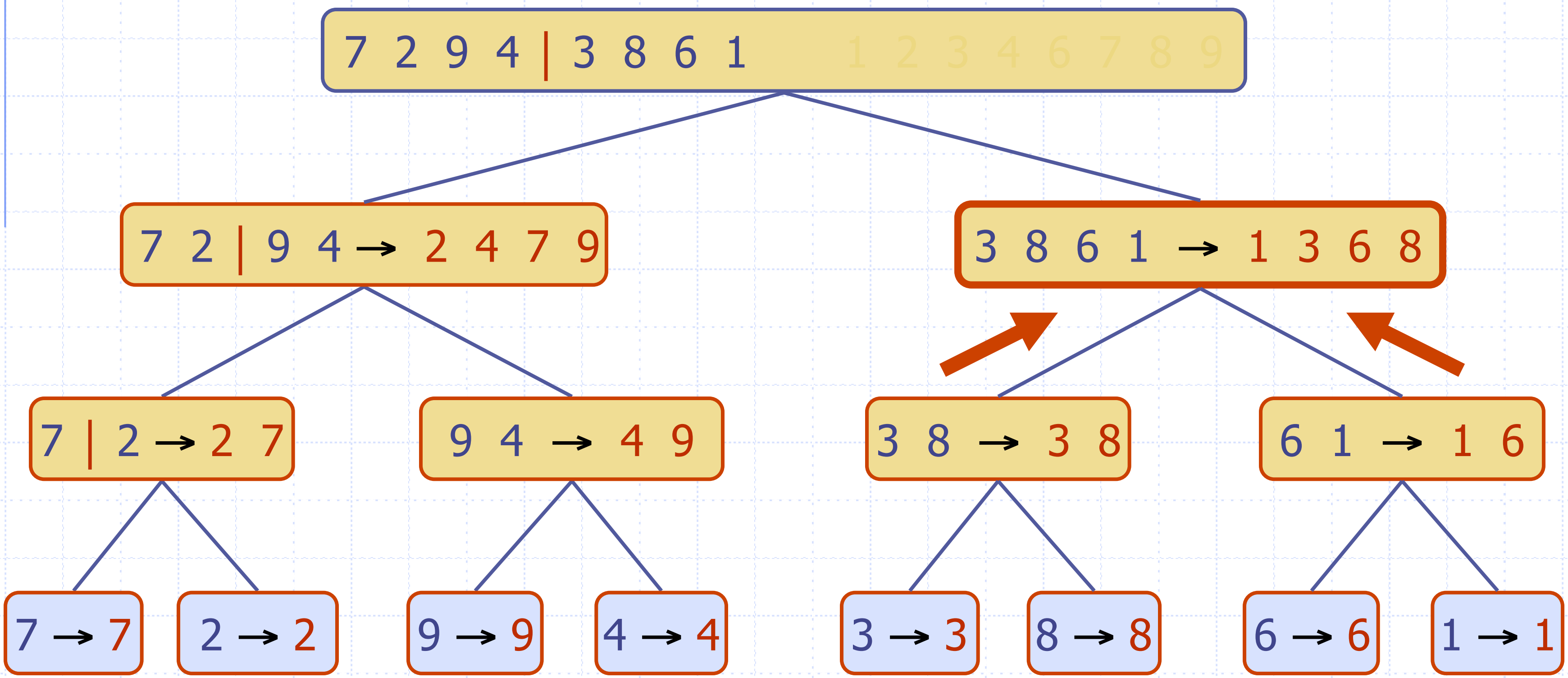
Execution Example (cont.)

- Merge



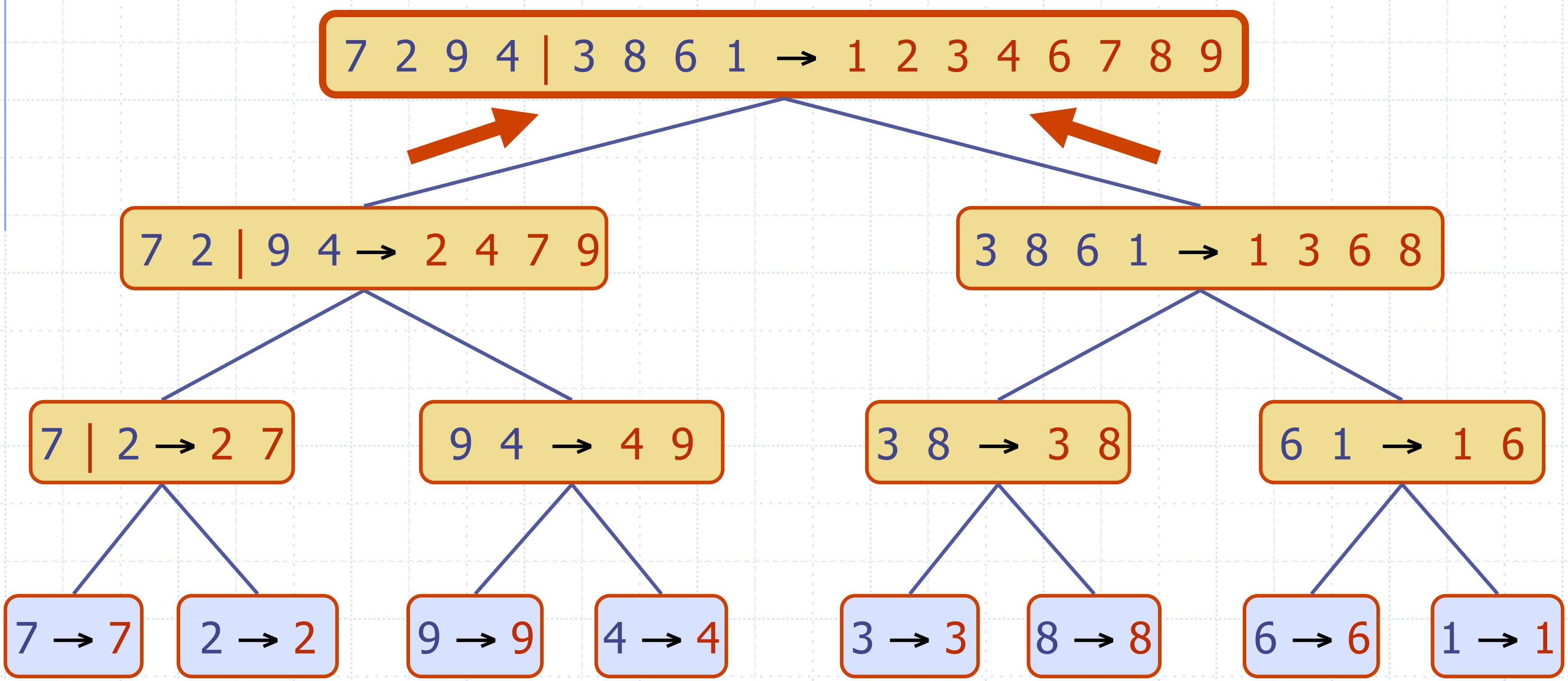
Execution Example (cont.)

- Recursive call, …, merge, merge



Execution Example (cont.)

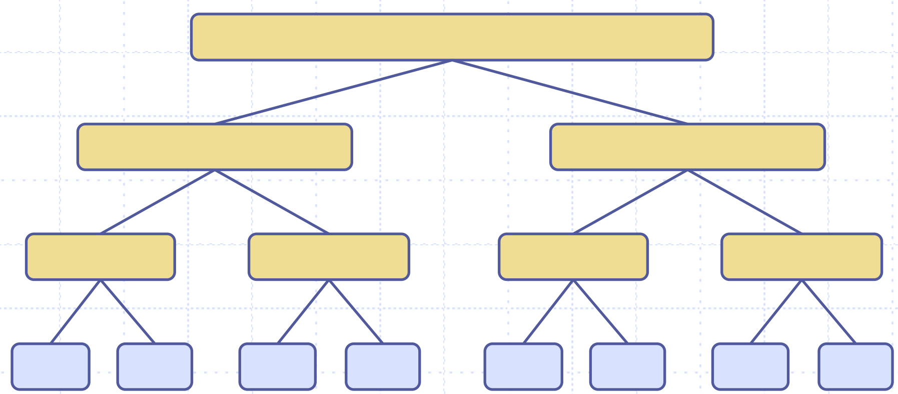
- Merge



Analysis of Merge-Sort

- The height h of the merge-sort tree is $O(\log n)$
  - at each recursive call we divide in half the sequence,
- The overall amount or work done at the nodes of depth i is $O(n)$
  - we partition and merge 2^i sequences of size $n/2^i$
  - we make 2^{i+1} recursive calls
- Thus, the total running time of merge-sort is $O(n \log n)$

depth	#seqs	size
0	1	n
1	2	$n/2$
i	2^i	$n/2^i$
…	…	…



Summary of Sorting Algorithms

Algorithm	Time	Notes
selection-sort	$O(n^2)$	▪ slow ▪ in-place ▪ for small data sets (< 1K)
insertion-sort	$O(n^2)$	▪ slow ▪ in-place ▪ for small data sets (< 1K)
heap-sort	$O(n \log n)$	▪ fast ▪ in-place ▪ for large data sets (1K — 1M)
merge-sort	$O(n \log n)$	▪ fast ▪ sequential data access ▪ for huge data sets (> 1M)