ICS 186A: Computer Graphics
Spring 2002
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Written Assignment 2
Assigned: Monday, April 15, 2002
Due: Friday, April 19, 2002
Estimated Time: 3 hrs

1. Consider a parallelepiped. A parallelepiped is a genus zero object with eight degree-three vertices and six parallelogram faces. (It is a sheared cuboid with no constraints on angle between the edges at a vertex.) Consider a 4x4 (rigid) model transformation matrix M. Naïve transformation of one vertex by M takes 16 multiplications and 12 additions/subtractions (4x4 matrix multiplied with a 4x1 vector). Hence for eight vertices, it would take 16x8 multiplications and 12x8 additions/subtractions. What is the minimum number of multiplications and additions/subtractions required to transform a parallelepiped?

2. Rasterize the line (P1,P2) where P1=(5,2), and P2=(15,8). Find also the color of each pixel rasterized by this line segment, given the color of P1 is 0.8 and that of P2 is 0.1. Also prove that the center of the pixel that is rasterized by this line is at most at a distance 0.5 from the actual line.

3. Consider a 2D square on the XY plane with side 2 units, the center at the origin and four sides parallel or perpendicular to the coordinate axes. Draw the picture of the transformed square after performing the following sequence of OpenGL commands. (Remember OpenGL post-multiplies the matrices in the order it is received, and finally the point is also post-multiplied.) (1.414 is the approximation of sqrt(2).)
   glRotatef(45,0,0,1);
   glTranslatef(1.414,0,0);
   glRotatef(45,0,0,1);
   Reduce the number of OpenGL function calls and thus give the new sequence of OpenGL function calls to effect the same transformation.

4. Consider the same square as in Question 3. Draw the picture of the transformed square after performing the following sequence of OpenGL operations.
   Case 1:
   glScalef(3,2,1);
   glTranslatef(2,2,0);
   Case 2:Draw the picture of the transformed square if the above operations were swapped.
   What should be the parameters of *glTranslatef* and *glScalef* in Case 2 so that the results of Case 2 and Case 1 are the same? Analyze the newly found parameters and their relationship with the parameters in Case 1.

5. The inverse R^{-1} of a rotation matrix R is its transpose R^T. What is the inter-relationship between different row vectors and column vectors of R? Use the definition and properties of inverse of matrices, and the interpretation of matrix multiplication in terms of dot product of vectors.

6. This question makes use of your understanding of Question 5. Consider the coordinate system due to general basis vectors X=(1,0,0),Y=(0,1,0), and Z=(0,0,1). Note the following facts about this system: this coordinate system has the origin (0,0,0), these basis vectors are unit vectors, these basis vectors are orthogonal to each other, and XxY=Z. Consider another *orthogonal* coordinate system U=(u1, u2, u3), V=(v1, v2, v3), and W=(w1,w2,w3) with the origin at (0,0,0). Assume that U, V, and W are unit vectors and UxV=W. Clearly there exists a rotation matrix R such that when the vectors U, V, and W are transformed using R, the vectors U, V, and W coincide with X, Y, and Z. Find the easiest way to compute R using all the above facts (and also your answer to Question 5). Your answer is the foundation for transforming a unit orthogonal coordinate system into the standard coordinate system with the same origin. [Hint: RxU=X, RxV=Y, RxW=Z. Put these three equations together as one single matrix equation and analyze this.]