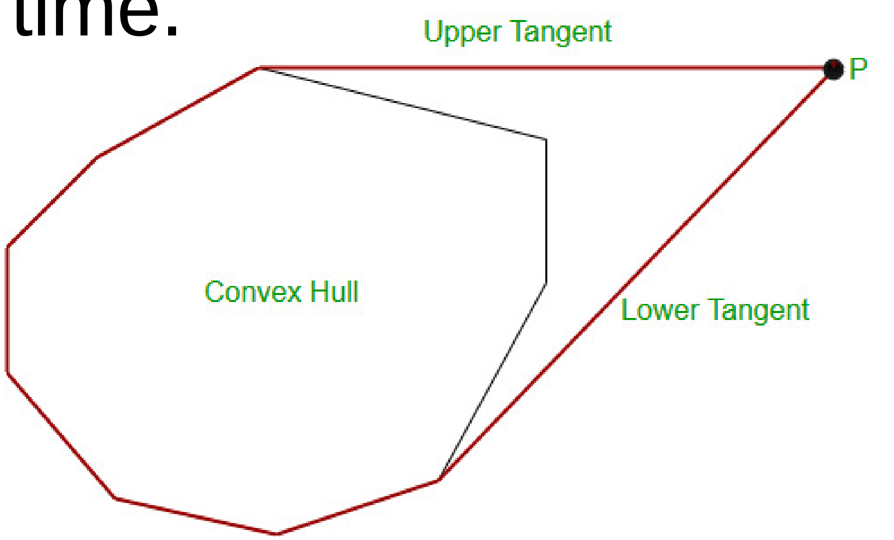
Chan's Convex Hull Algorithm

Michael T. Goodrich

UNIVERSITY of CALIFORNIA IRVINE

Convex Hull Binary Search

- There is a binary search method for finding the common upper tangent for two convex hulls separated by a line in O(log n) time.
- This same method also works to find the upper tangent between a point and a convex polygon in O(log n) time.



Review

- The upper-hull plane-sweep algorithm runs in **O(n log n) time.**
  - This algorithm is sometimes called “Graham Scan”
- The Gift Wrapping algorithm runs in **O(nh) time**, where h is the size of the hull.
  - This algorithm is sometimes called “Jarvis March”
- Which of these is best depends on h
- It would be nice to have one optimal algorithm for all values of h…

Discrete Comput Geom 16:361–368 (1996)

Discrete & Computational
Geometry

Optimal Output-Sensitive Convex Hull Algorithms in Two and Three Dimensions*

T. M. Chan

Department of Computer Science, University of British Columbia,
Vancouver, British Columbia, Canada V6T 1Z4

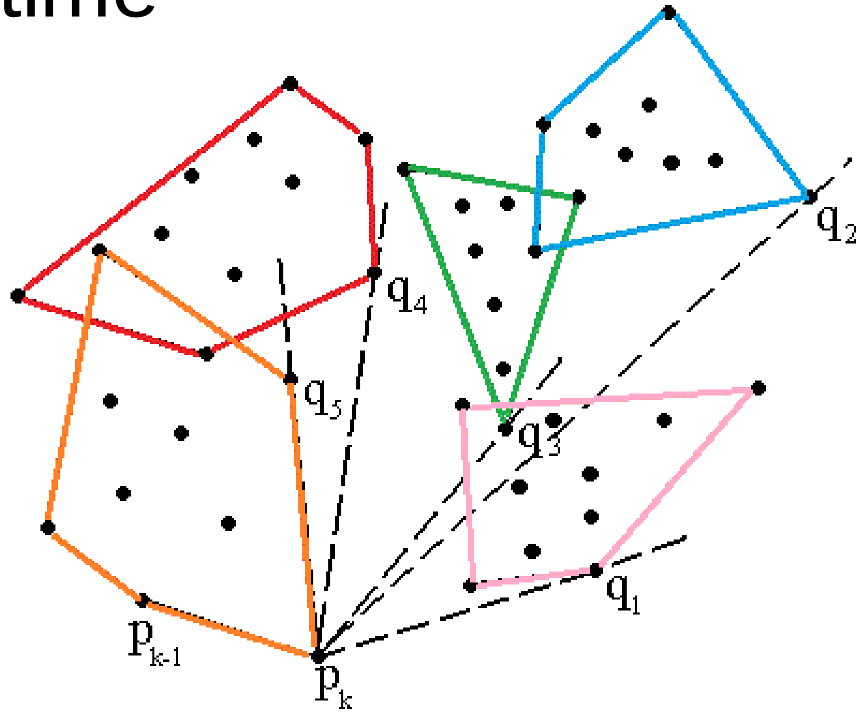
Abstract. We present simple output-sensitive algorithms that construct the convex hull of a set of n points in two or three dimensions in worst-case optimal $O(n \log h)$ time and $O(n)$ space, where h denotes the number of vertices of the convex hull.

Main Idea

- Assume, for now, we have an estimate, m, that is O(h).
- Divide our set into n/m groups of size O(m) each
- Find the convex hull of each group in O(m log m) time using Graham scan
- Next, do a Jarvis march around all these “mini hulls.”

Jarvis March Steps

- Start with a point, p_k, on the convex hull
- Find the tangent for every mini hull with p_k
- Takes O((n/m)log m) time
- Pick the furthest one
- Repeat



Analysis

- Doing all the Graham scans to build the mini hulls takes O((n/m)m log m) = O(n log m) time.
- Doing each Jarvis march step takes O((n/m) log m) time. There are h <= m such steps to find the convex hull. So all these steps take O(n log m) time.
- If m is O(h), the running time is O(n log h).
- But we don't know h…

Pseudo Code

Algorithm Hull2D(P, m, H), where $P \subset E^2$, $3 \leq m \leq n$, and $H \geq 1$

1. partition P into subsets $P_1, \ldots, P_{\lceil n/m \rceil}$ each of size at most m
2. for $i = 1, \ldots, \lceil n/m \rceil$ do
3.   compute $\text{conv}(P_i)$ by Graham's scan and store its vertices in an array in ccw order
4. $p_0 \leftarrow (0, -\infty)$
5. $p_1 \leftarrow$ the rightmost point of P
6. for $k = 1, \ldots, H$ do
7.   for $i = 1, \ldots, \lceil n/m \rceil$ do
8.   compute the point $q_i \in P_i$ that maximizes $\angle p_{k-1} p_k q_i$ $(q_i \neq p_k)$ by performing a binary search on the vertices of $\text{conv}(P_i)$
9. $p_{k+1} \leftarrow$ the point q from $\{q_1, \ldots, q_{\lceil n/m \rceil}\}$ that maximizes $\angle p_{k-1} p_k q$
10.   if $p_{k+1} = p_1$ then return the list $\langle p_1, \ldots, p_k \rangle$
11. return *incomplete*

Guessing an estimate for h

- Start with m = 4.
- Run Chan’s algorithm. If it doesn’t return *incomplete*, we’re done.
- Otherwise, try again with $m = m^2$.
- Keep repeating this until we get a complete hull.

The Complete Running Time

- The complete running time (adding up the terms in reverse order):

$$O(n \log h + n \log h^{1/2} + n \log h^{1/4} + \ldots)$$
$$= O(n \log h + (1/2)n \log h + (1/4)n \log h + \ldots)$$
$$= O(n \log h).$$