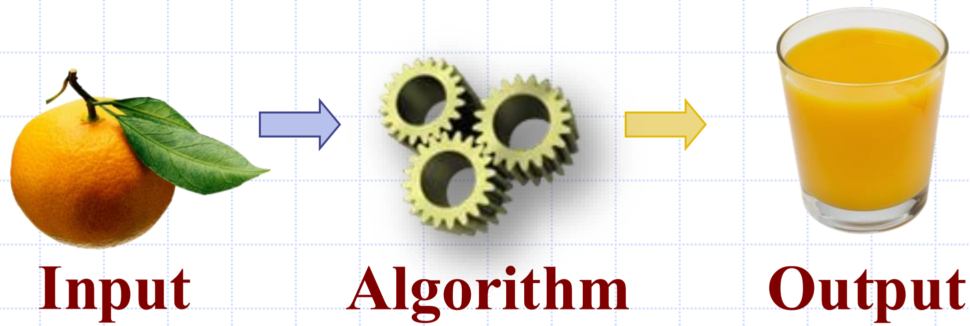
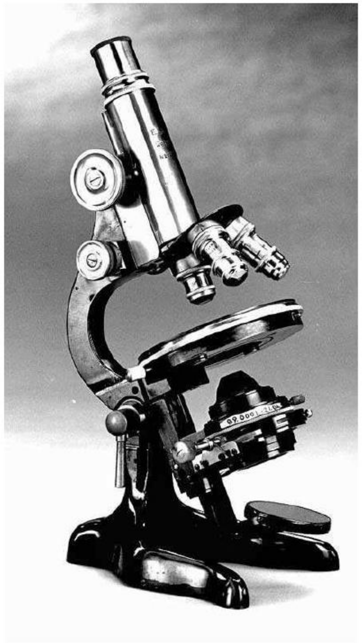
Algorithm Analysis



Scalability

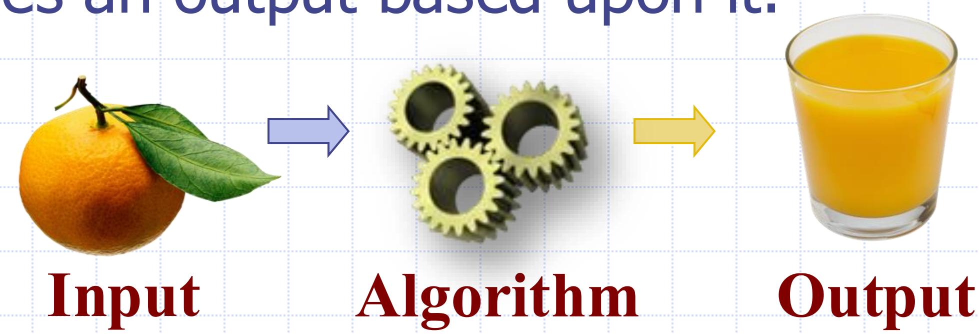
- Scientists often have to deal with differences in scale, from the microscopically small to the astronomically large.
- Computer scientists must also deal with scale, but they deal with it primarily in terms of data volume rather than physical object size.
- **Scalability** refers to the ability of a system to gracefully accommodate growing sizes of inputs or amounts of workload.



Microscope: U.S. government image, from the N.I.H. Medical Instrument Gallery, DeWitt Stetten, Jr., Museum of Medical Research. Hubble Space Telescope: U.S. government image, from NASA, STS-125 Crew, May 25, 2009.

Algorithms and Data Structures

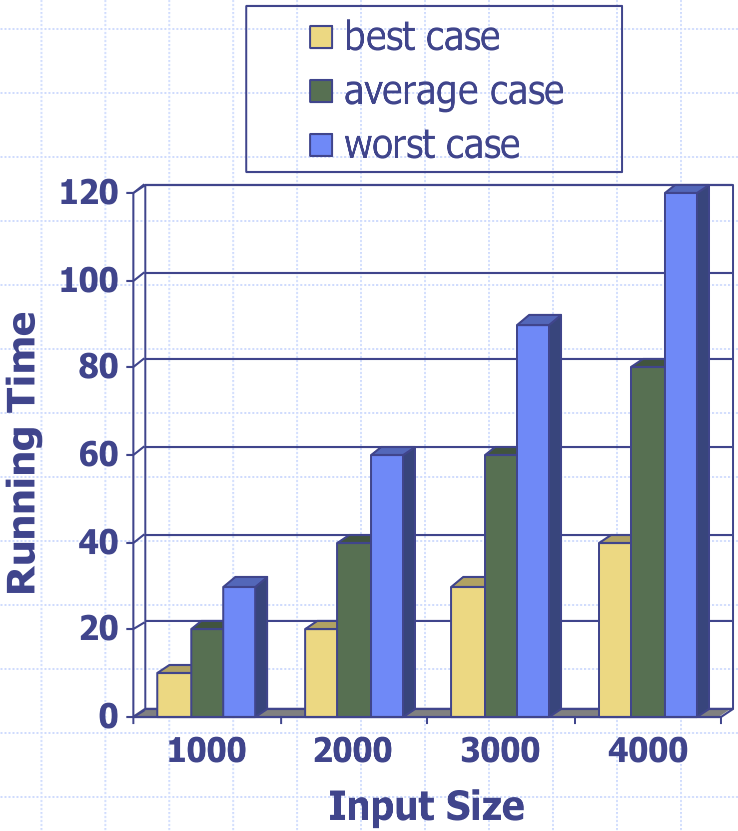
- An **algorithm** is a step-by-step procedure for performing some task in a finite amount of time.
  - Typically, an algorithm takes input data and produces an output based upon it.



- A **data structure** is a systematic way of organizing and accessing data.

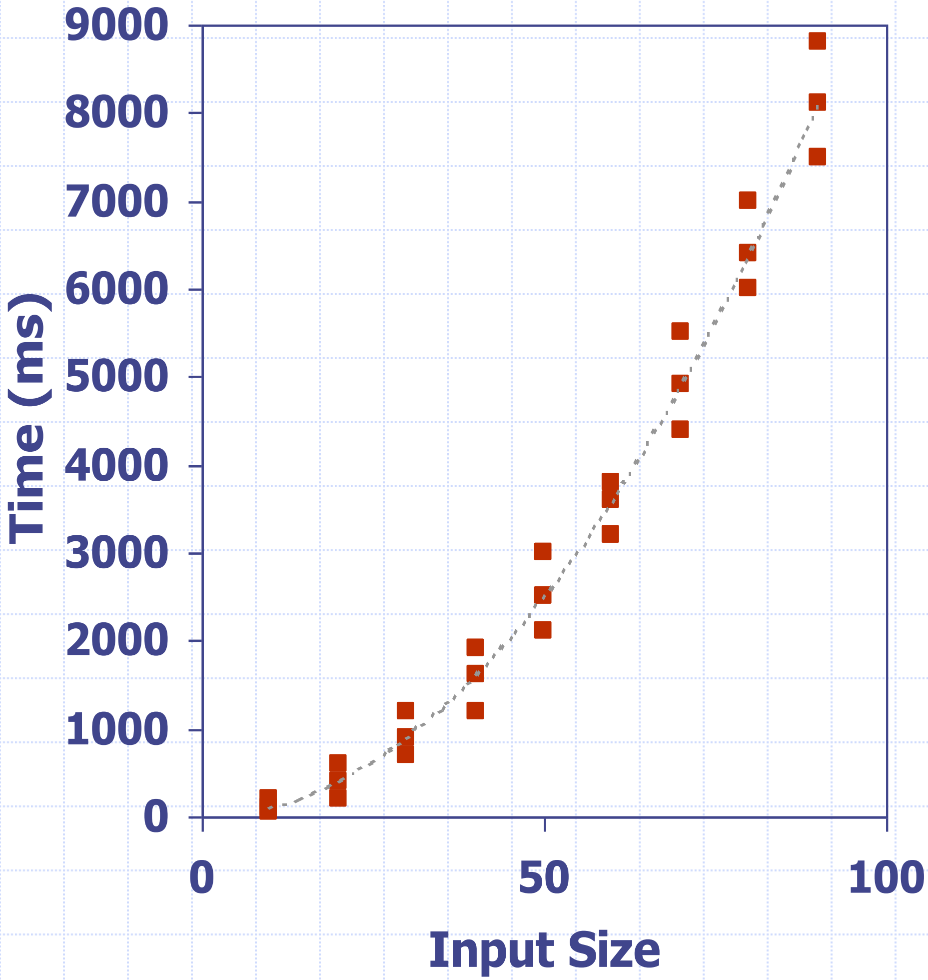
Running Times

- Most algorithms transform input objects into output objects.
- The running time of an algorithm typically grows with the input size.
- Average case time is often difficult to determine.
- We focus primarily on the **worst case running time**.
  - Theoretical analysis
  - Might not capture real-world performance



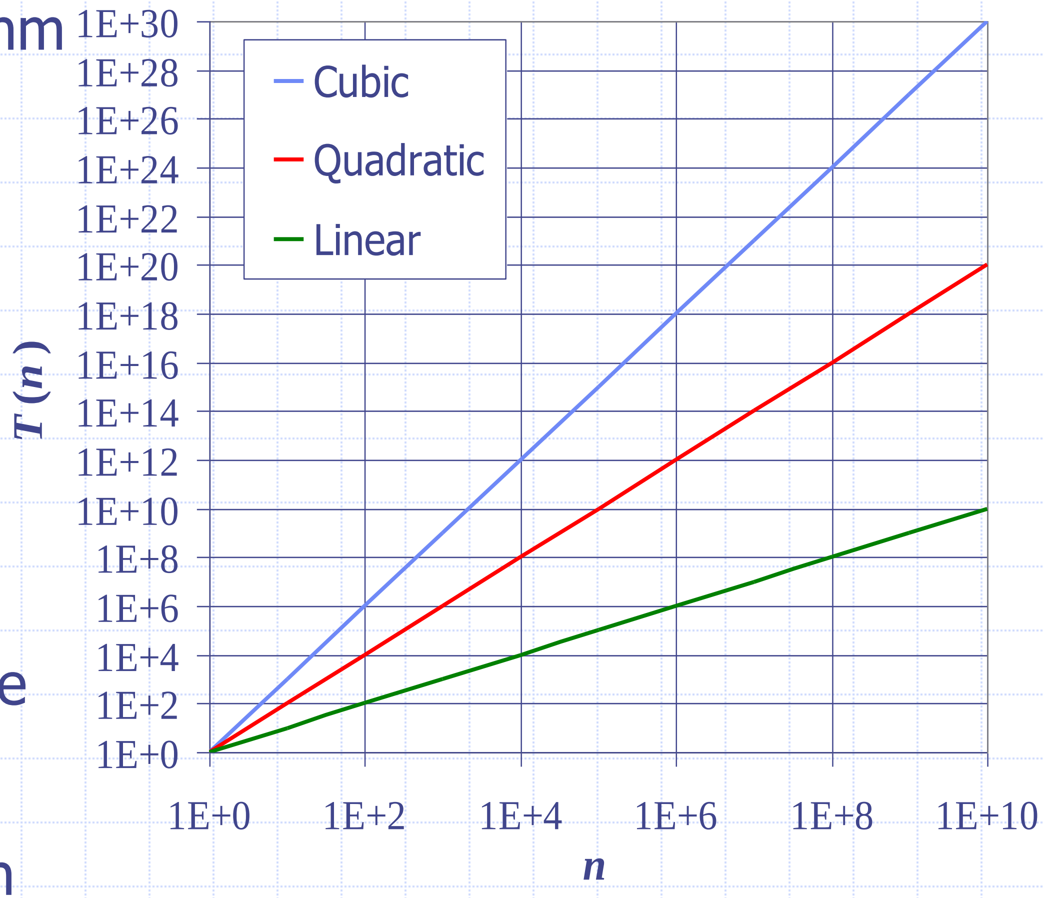
Experimental Studies

- Write a program implementing the algorithm
- Run the program with inputs of varying size and composition, noting the time needed:
- Plot the results
- Try to match a curve to the times



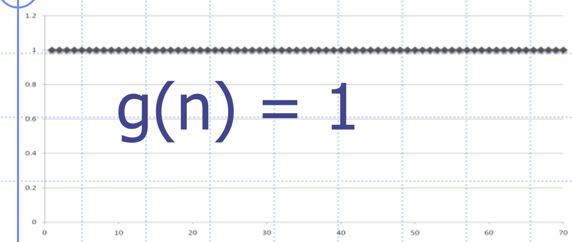
Seven Important Functions

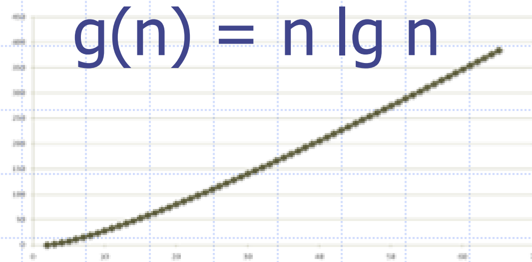
- Seven functions that often appear in algorithm analysis:
  - Constant ≈ $\mathbf{1}$
  - Logarithmic ≈ log $\boldsymbol{n}$
  - Linear ≈ $\boldsymbol{n}$
  - N-Log-N ≈ $\boldsymbol{n}$ log $\boldsymbol{n}$
  - Quadratic ≈ $\boldsymbol{n}^2$
  - Cubic ≈ $\boldsymbol{n}^3$
  - Exponential ≈ 2^n
- In a **log-log chart**, the slope of the line corresponds to the exponent in the growth rate

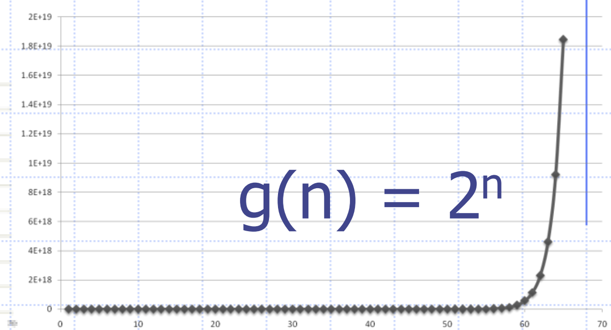


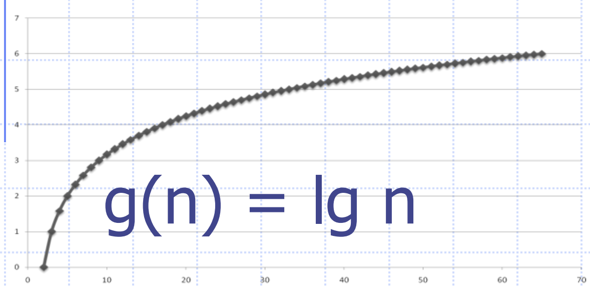
Functions Graphed Using “Normal” Scale

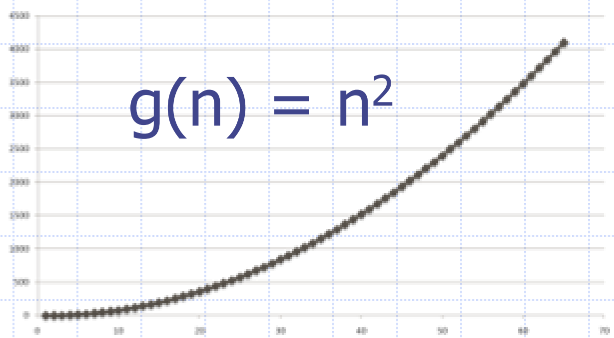
Slide by Matt Stallmann included with permission.

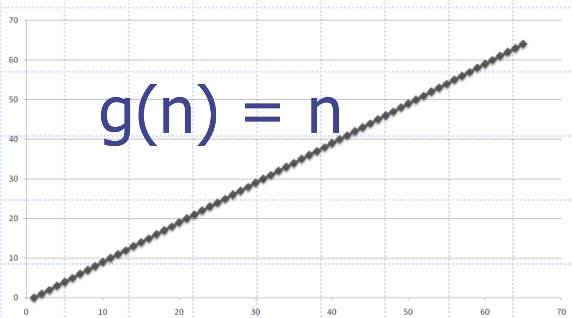


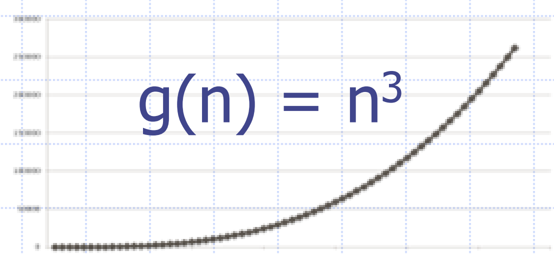












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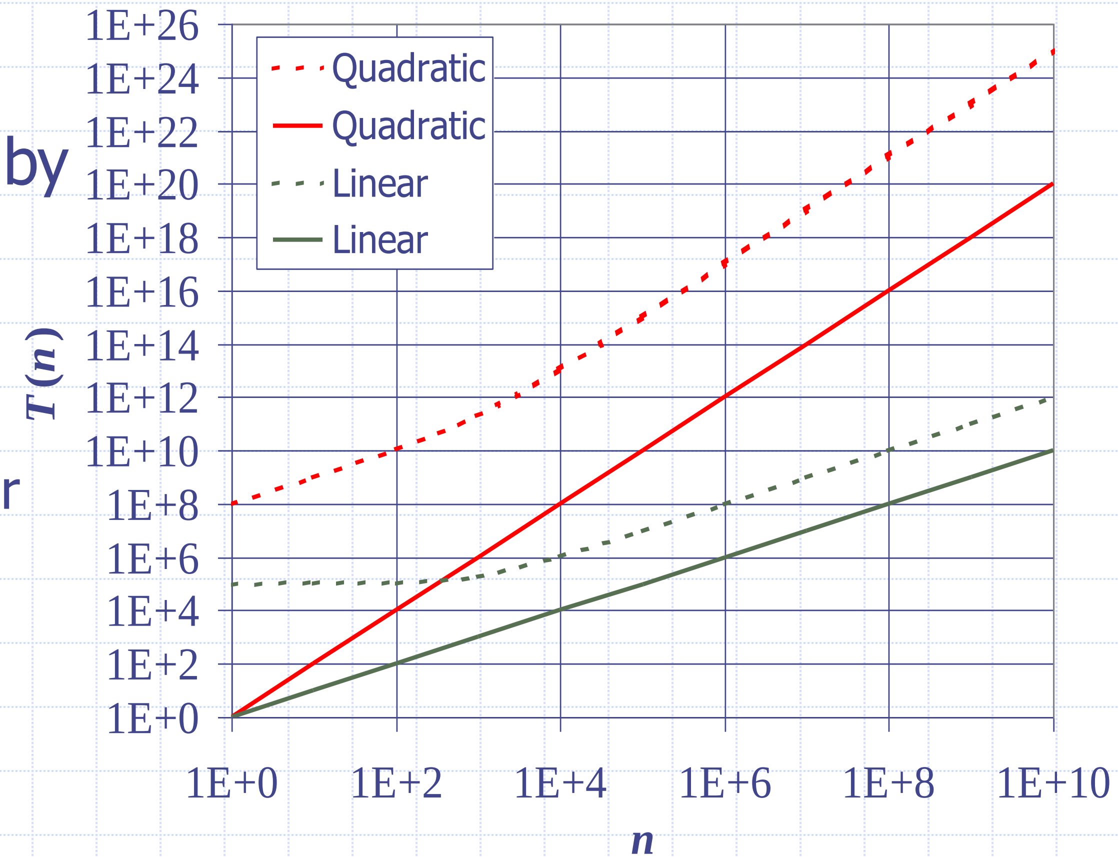
Why Growth Rate Matters

if runtime is...	time for n + 1	time for 2 n	time for 4 n
c lg n	c lg (n + 1)	c (lg n + 1)	c(lg n + 2)
c n	c (n + 1)	2c n	4c n
c n lg n	~ c n lg n + c n	2c n lg n + 2cn	4c n lg n + 4cn
c n^2	~ c n^2 + 2c n	**4c n^2**	16c n^2
c n^3	~ c n^3 + 3c n^2	8c n^3	64c n^3
c 2^n	c 2^{n+1}	c 2^{2n}	c 2^{4n}

runtime quadruples when problem size doubles

Constant Factors

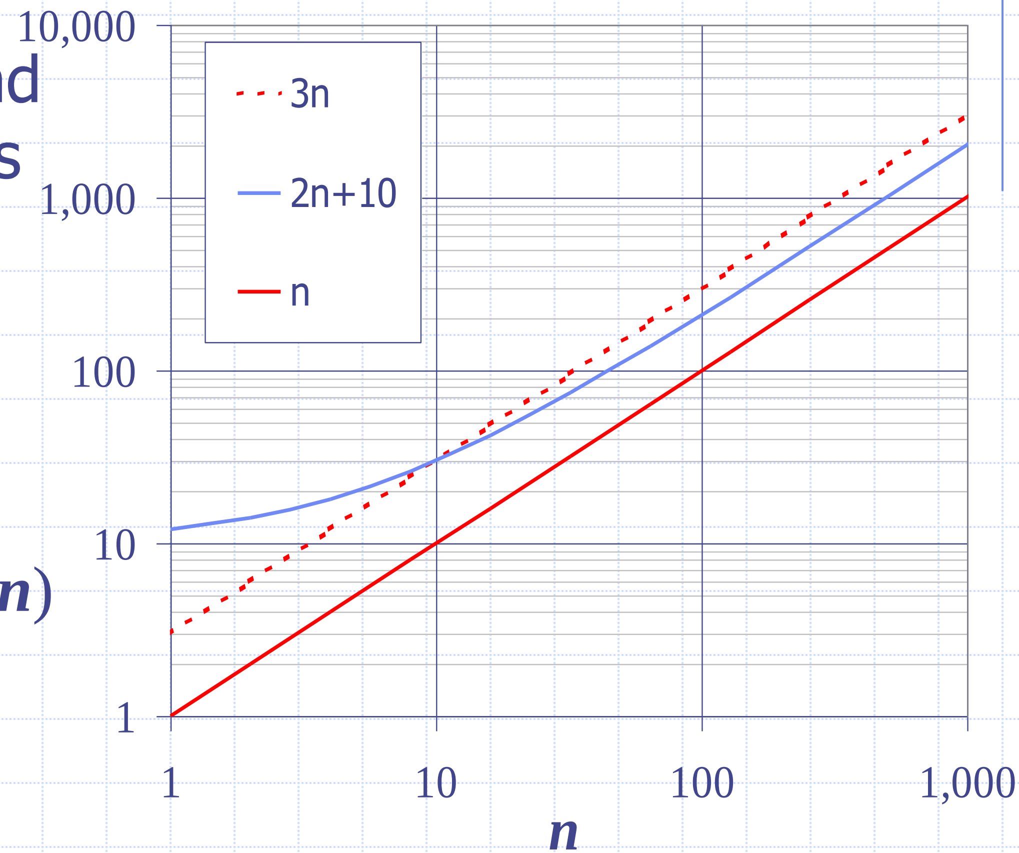
- The growth rate is minimally affected by
  - constant factors or
  - lower-order terms
- Examples
  - $10^2 n + 10^5$ is a linear function
  - $10^5 n^2 + 10^8 n$ is a quadratic function



Big-Oh Notation

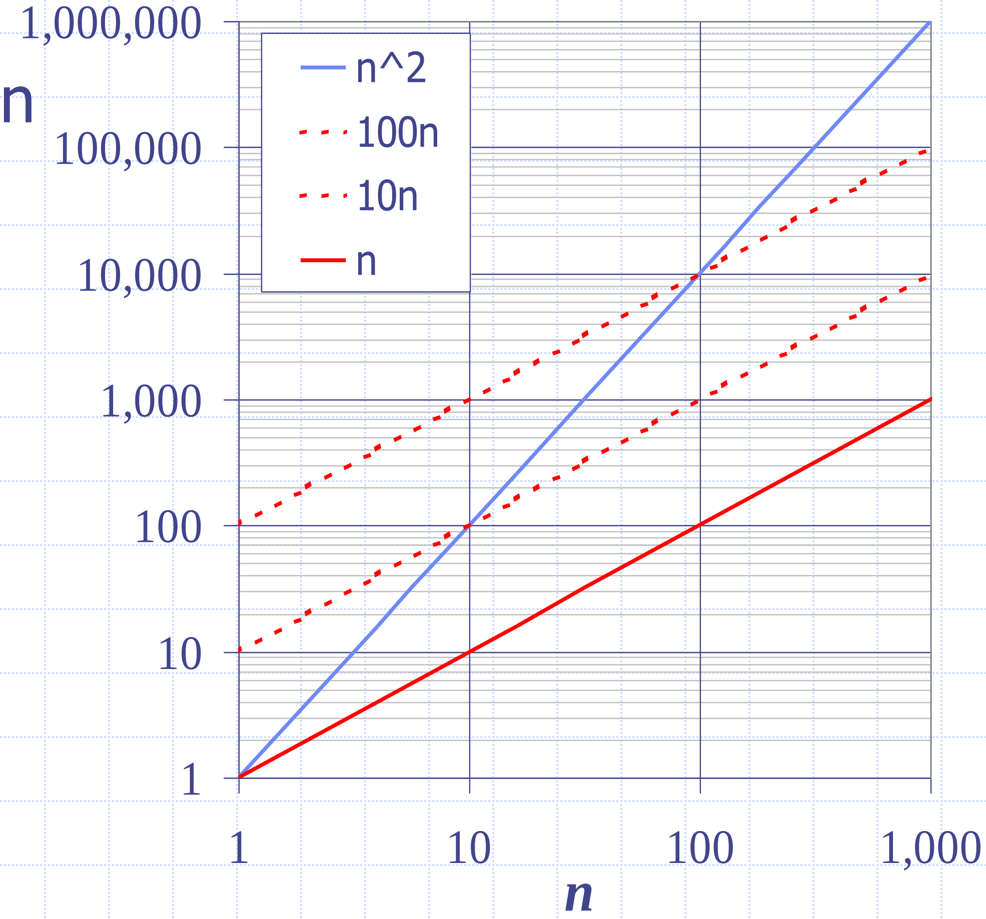
- Given functions $f(n)$ and $g(n)$, we say that $f(n)$ is $O(g(n))$ if there are positive constants c and n_0 such that

  $f(n) \leq cg(n)$ for $n \geq n_0$
- Example: $2n + 10$ is $O(n)$
  - $2n + 10 \leq cn$
  - $(c - 2)\, n \geq 10$
  - $n \geq 10/(c - 2)$
  - Pick $c = 3$ and $n_0 = 10$



Big-Oh Example

- Example: the function n^2 is not $O(n)$
  - $n^2 \leq cn$
  - $n \leq c$
  - The above inequality cannot be satisfied since c must be a constant



Big-Oh Rules

- If is $f(n)$ a polynomial of degree d, then $f(n)$ is $O(n^d)$, i.e.,
  1. Drop lower-order terms
  2. Drop constant factors
- Use the smallest possible class of functions
  - Say "$2n$ is $O(n)$" instead of "$2n$ is $O(n^2)$"
- Use the simplest expression of the class
  - Say "$3n + 5$ is $O(n)$" instead of "$3n + 5$ is $O(3n)$"

Relatives of Big-Oh

big-Omega

- f(n) is Ω(g(n)) if there is a constant c > 0 and an integer constant $n_0 \geq 1$ such that

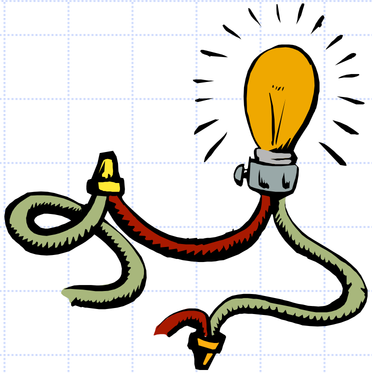
$$f(n) \geq c\, g(n) \text{ for } n \geq n_0$$

big-Theta

- f(n) is Θ(g(n)) if there are constants c′ > 0 and c″ > 0 and an integer constant $n_0 \geq 1$ such that

$$c'g(n) \leq f(n) \leq c''g(n) \text{ for } n \geq n_0$$

Intuition for Asymptotic Notation



big-Oh

- f(n) is O(g(n)) if f(n) is asymptotically less than or equal to g(n)

big-Omega

- f(n) is Ω(g(n)) if f(n) is asymptotically greater than or equal to g(n)

big-Theta

- f(n) is Θ(g(n)) if f(n) is asymptotically equal to g(n)